MATH 133 – Calculus with Fundamentals 1
Practice on Functions, Graphs, Shifting and Scaling
September 7, 2015

Background

Recall from the video "lecture" for last Friday that a *function* f from a *domain* D to a set Y is a rule that assigns to each element x in D, a unique element $f(x)$ in Y. This means that the graph of f passes the "vertical line test:" for each x in D, there is exactly one point $(x, f(x))$ on the graph of f. The set of values $f(x)$ for x in D is called the *range* of the function. The range is a subset of Y (it can be all of Y but it does not have to be). (Almost) all of the functions we will see have subsets of the real numbers as their domains and ranges.

Questions

1) The question refers to the plot of temperature versus time over a 24-hour period at a particular location that we saw Friday (see back of this page). Is this the graph of a function? If so, what are the domain and range, as intervals? If not, why does it fail to be a function?

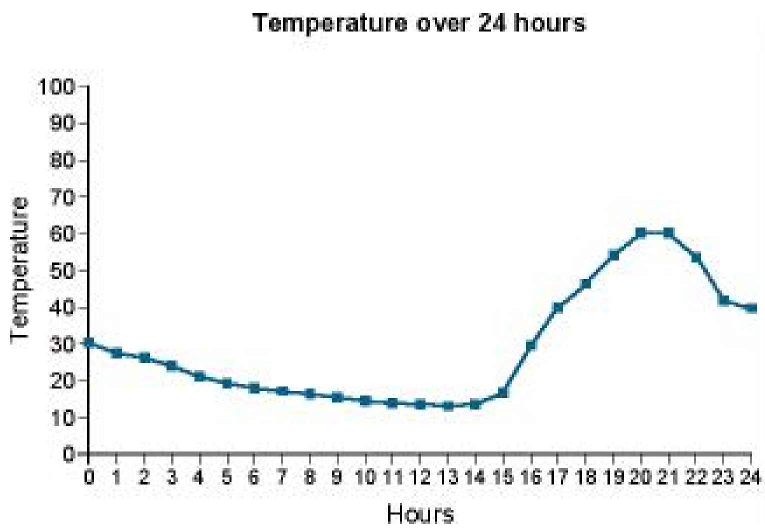
2) Look at Figure 26 on page 10 of our text book. Which of these graphs is the graph of a function? Explain.

3) If no domain is specified, for a function defined by a formula, then the "rule of thumb" is to take the domain to be the set of all real x such that the formula gives a well-defined value. Using this,

   (a) What is the domain of the function defined by $f(x) = \frac{1}{x^2-4}$?
   (b) What is the domain of the function defined by $f(x) = \sqrt{4+x}$?

4) All parts of this question deal with $f(x) = x^3$.

   (a) Sketch the graph $y = f(x)$ on the domain $[-2, 2]$
   (b) Sketch the graph $y = f(x) - 3$ on the same domain as in part (a).
   (c) Sketch the graph $y = f(x + 1)$ on the domain $[-3, 1]$ using your graph in part (a)
   (d) Sketch the graph $y = 2f(x)$ on the domain $[-2, 2]$.
   (e) Sketch the graph $y = f(2x)$ on the domain $[-1, 1]$. How is this graph related to $y = f(x)$?
   (f) Sketch the graph $y = f\left(\frac{x}{2}\right)$ on the domain $[-4, 4]$. How is this graph related to $y = f(x)$?
   (g) (Practice on finding patterns) From section 1.1 of the text and videos, we know $y = cf(x)$ is a vertical stretching or compression of $y = f(x)$. What is the corresponding description of the graph $y = f(cx)$? How does this depend on the size of c? (Hint: Try drawing the graphs $y = f(x) = x^3$ on $[-2, 2]$ and the graphs from part (e) and part (f) with those given domains, together on one set of axes. Note that you'll need to take x in the interval $[-4, 4]$ to get all of them to "fit" but draw the graphs exactly as you did before, i.e. on the domains as stated before.)



Hours	Temperature
0	30.7
1	27.7
2	26.5
3	24
4	21.4
5	19.5
6	18.2
7	17.4
8	16.8
9	15.7
10	14.6
11	14
12	13.6
13	13.3
14	13.7
15	16.7
16	29.7
17	40.3
18	46.6
19	54.6
20	60.6
21	60.6
22	53.9
23	42
24	40

Figure 1: Figure for Question 1