MATH 133 – Calculus with Fundamentals 1
The Derivative Function
October 20, 2015

Background

Recall that last time, we introduced the *derivative* of a function f at x:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists.

In today's video we introduced the idea of deriving formulas for $f'(x)$ for a general x, and considering f' as a new function in its own right. We said:

(1) If $f(x) = x^n$ for any number n, then $f'(x) = nx^{n-1}$.

(2) If $f'(x)$ exists, then so does $(kf)'(x)$ and $(kf)'(x) = kf'(x)$.

(3) If $f'(x)$ and $g'(x)$ both exist, then so does $(f+g)'(x)$, and $(f+g)'(x) = f'(x) + g'(x)$.

Questions

(1) Compute $f'(x)$ for $f(x) = x^{5/3}$ using point (1) in the Background above.

(2) Compute $f'(x)$ for $f(x) = x^4 + 5x^2 + 7 + 2x^{-3}$ using points (1), (2), and (3) in the Background above.

(3) Make up your own example function of the same type as the one in (2) and compute $f'(x)$ using points (1), (2), and (3) in the Background.

(4) What is $f'(x)$ if $f(x) = \pi^\pi$? (Be careful!)

The video for today gave the idea for showing point (1) in the Background when n is a positive integer, and stated that the same formula works for all n. Let's verify this in some special cases.

(5) Verify the formula in point (1) for $f(x) = x^4$. That is, you want to show $f'(x) = 4x^3$ using the definition of $f'(x)$.

(6) Verify the formula in point (1) for $f(x) = \sqrt{x} = x^{1/2}$. That is, you want to show $f'(x) = \frac{1}{2}x^{-1/2}$ using the definition of $f'(x)$.

(7) Verify the formula in point (1) for $f(x) = \frac{1}{x^2} = x^{-2}$. That is, you want to show $f'(x) = -2x^{-3}$ using the definition of $f'(x)$.

(8) Why does point (2) in the background work? Explain in terms of the limit definition, and in terms of how the graphs $y = f(x)$ and $y = kf(x)$ are related.

Finally, it is important to realize that the limit in the definition of $f'(x)$ can fail to exist at some of the x in the domain of f.

(9) Consider $y = |x|$ (the absolute value function). Does

$$f'(0) = \lim_{h \to 0} \frac{|h| - 0}{h}$$

exist? Why or why not? Does this make sense in terms of the graph $y = |x|$?