MATH 133 – Calculus with Fundamentals 1
The Derivative of a Function
October 19, 2015

Background

We are now ready to begin Chapter 3 in our textbook. In the video for today's class, we introduced the *derivative* of a function f at $x = a$ in the domain of f:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a},$$

provided the limit exists. If the limit does exist, then by what we said in Section 2.1, $f'(a)$ will give the slope of the tangent line to the graph $y = f(x)$ at the point $(a, f(a))$. If x represents time, and $f(x)$ is a position, then $f'(a)$ would be the instantaneous velocity.

All the techniques we learned in Chapter 2 for computing indeterminate form limits were, in fact, *set up to compute the limits giving* $f'(a)$(!) Let's practice (and review) some of those techniques!

Questions

(1) Compute $f'(1)$ for $f(x) = x^3 + 2x + 1$ and use your result to find the equation of the tangent line to the graph $y = x^3 + 2x + 1$ at the point $(1, 4)$.

(2) Compute $f'(3)$ for $f(x) = \sqrt{x+1}$ and use your result to find the equation of the tangent line to the graph $y = \sqrt{x+1}$ at the point $(3, 2)$.

(3) Compute $f'(2)$ for $f(x) = \frac{1}{x}$ and use your result to find the equation of the tangent line to the graph $y = \frac{1}{x}$ at the point $(2, 1/2)$.

We will now concentrate on finding *general formulas* for derivatives.

(4) Adapt what you did in question (1) above to compute $f'(a)$–the derivative at a general $x = a$ for $f(x) = x^3 + 2x + 1$.

(5) Adapt what you did in question (2) above to compute $f'(a)$–the derivative at a general $x = a$ for $f(x) = \sqrt{x+1}$. Here there is a restriction on which a "work." What is that restriction? Does this make sense, thinking of the graph $y = \sqrt{x+1}$? (Note: this is part of the parabola with equation $x = y^2 - 1$.)

(6) Adapt what you did in question (3) above to compute $f'(a)$–the derivative at a general $x = a$ for $f(x) = \frac{1}{x}$. Does your formula make sense, thinking of the shape of the graph $y = \frac{1}{x}$? In particular, what is true about $f'(a)$ if a is very close to zero? And what about a very large in absolute value?