MATH 133 – Calculus with Fundamentals 1
Trigonometric Limits
October 8, 2015

Background

In today's video, we saw an additional technique for evaluating limits called the "Squeeze Theorem" and the very important limit:

$$\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1. \tag{1}$$

Recall that the Squeeze Theorem says: Assume $l(x) \leq f(x) \leq u(x)$ on some interval containing $x = c$ (except possibly at $x = c$) and $\lim_{x \to c} l(x) = \lim_{x \to c} u(x) = L$ (we might say "f is squeezed by l and u at $x = c$" to describe this). Then $\lim_{x \to c} f(x)$ exists and equals L as well.

Questions

Do the following problems from Section 2.6 in our text:

(1) Exercise 1 – in particular, what is $\lim_{x \to 1} f(x)$ and how do we know that?

(2) Exercise 4

(3) Exercise 5 – Note that the question concerns $\lim_{x \to 1} f(x)$.

Evaluate the following limits using (1):

(4)
$$\lim_{t \to 0} \frac{\sin(t)}{8t}.$$

(5)
$$\lim_{t \to 0} \frac{\sin(8t)}{t}.$$

(Hint: For this one let $u = 8t$ and convert the t in the denominator to an equivalent expression in terms of u. Note that $t \to 0$ implies $u = 8t \to 0$ also.)

(6)
$$\lim_{t \to 0} \frac{\sin(3t)}{\sin(5t)}.$$

(Hint: Rewrite as follows:

$$\frac{\sin(3t)}{\sin(5t)} = \frac{\frac{\sin(3t)}{t}}{\frac{\sin(5t)}{t}},$$

then proceed as in question (5).