MATH 133 – Calculus with Fundamentals 1
Practice on Absolute Values and Intervals
September 4, 2015

*Background*

Recall from the video "lecture" for today that the absolute value of a real number is defined as

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

Also, we have several different types of *intervals* in the real number system written as follows:

$$(a, b) = \{x \mid a < x < b\} \tag{1}$$
$$[a, b] = \{x \mid a \leq x \leq b\} \tag{2}$$
$$[a, b) = \{x \mid a \leq x < b\} \tag{3}$$
$$(a, b] = \{x \mid a < x \leq b\}. \tag{4}$$

Today we will practice on some problems related to these ideas.

*Questions*

0) (Warm-up) Real numbers can all be expressed by (possibly infinite or non-repeating) decimal expansions. But exactly what does a decimal expansion like

$$\pi = 3.1415926\cdots$$

*mean*? What does a finite decimal expansion like $\frac{15}{8} = 1.875$ mean? (Hint: It's really a way of writing the number as a sum of fractions with denominators of a particular form.)

1) The set of real numbers $x$ satisfying $|x| < 17$ is an interval. Write it using the appropriate interval notation.

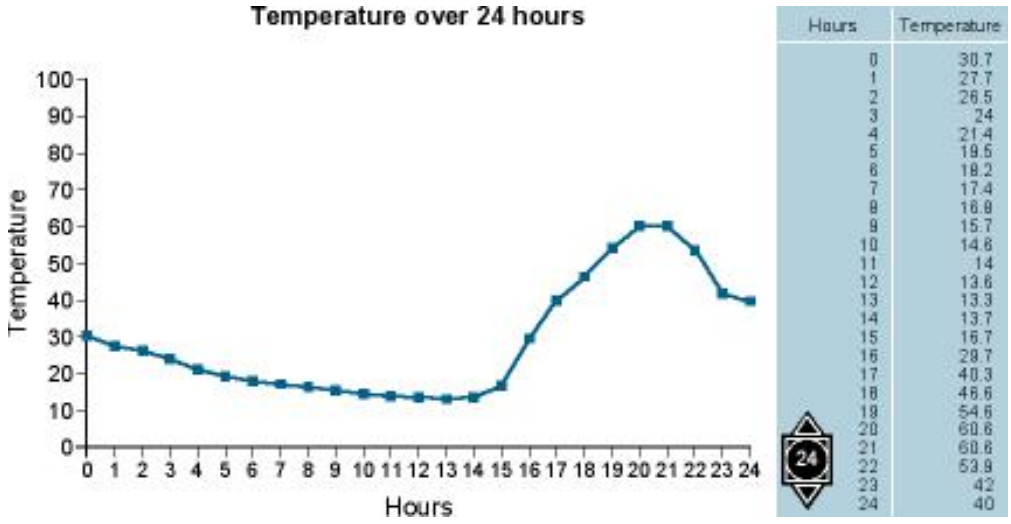
2) The set of real numbers $x$ satisfying $|x - 3| \leq 1$ is also an interval. Write it using the appropriate interval notation.

3) The set of real numbers $x$ satisfying $|3x + 1| < 4$ is also an interval. Write it using the appropriate interval notation.

4) Write the interval $(5, 9)$ in the form: "the set of all $x$ such that $|x - c| < r$" for some real numbers $c$ and $r$.

5) Same as question 4, but for the interval $\left[-\frac{2}{5}, 3\right]$.

6) Using the graph of temperature versus time over a 24-hour period at a particular location, on the back of this page answer these questions:




| Hours | Temperature |
|---|---|
| 0 | 30.7 |
| 1 | 27.7 |
| 2 | 26.5 |
| 3 | 24 |
| 4 | 21.4 |
| 5 | 19.5 |
| 6 | 18.2 |
| 7 | 17.4 |
| 8 | 16.8 |
| 9 | 15.7 |
| 10 | 14.6 |
| 11 | 14 |
| 12 | 13.6 |
| 13 | 13.3 |
| 14 | 13.7 |
| 15 | 16.7 |
| 16 | 28.7 |
| 17 | 40.3 |
| 18 | 46.6 |
| 19 | 54.6 |
| 20 | 60.6 |
| 21 | 60.6 |
| 22 | 53.9 |
| 23 | 42 |
| 24 | 40 |

Figure 1: Figure for Question 6

(a) Write the collection of all *temperatures* measured over the 24 hour period as an interval $[a, b]$ for suitable temperature values $a, b$ in degrees Fahrenheit. (There's an unstated assumption behind this question. Can you see what that is? Does it seem reasonable to make that assumption?)

(b) Estimate the interval $[c, d]$ of *times* in hours for which the temperature was 16.8 degrees or less.

(c) At how many different times does it appear that the temperature was exactly 40 degrees F?

Note: The temperature values at the start of each hour are given in the table at the right.