

MATH 133 – Calculus with Fundamentals 1  
Applied Optimization Problems  
December 4 and 7, 2015

*Background*

Yesterday, we discussed some first examples of optimization or “max/min” problems. Many of these come from more or less realistic applications. A general strategy for solving these is:

- Step 1: Draw pictures illustrating several different possible solutions of the problem, if appropriate.
- Step 2: Identify which quantities are changing in your different solutions (the variables), and name them. Write down the quantity to be maximized or minimized, giving it a name too.
- Step 3: (If there is more than one variable), write down any relations between the variables, and use the relation to solve for all variables in terms of one of them.
- Step 4: Substitute from results of Step 3 into the function giving the quantity from Step 2. The goal here is to obtain a function of *only one* variable.
- Step 5: Find the critical points of the function from Step 4.
- Step 6. Classify critical critical as local maxima or local minima using First or Second Derivative Tests. If the variable is limited to an interval, determine the function values at the endpoints, and find the overall maximum or minimum as indicated in the problem. (Note: “largest, biggest, greatest, etc.” in the statement of the problem usually means you are looking for a maximum value, while “smallest, least, cheapest, etc.” usually indicates you are looking for a minimum.)
- Step 7. Find the maximum or minimum and write down the final answer. (And, of course, be sure you are answering the question that was asked!)

Today and next time, we want to practice using this on several examples.

*Discussion Questions*

- A) Daphnis Aipolos, a Greek goatherd, wants to fence in a rectangular pasture for his prize goats. He has 1600 meters of fencing to use, and he can use the straight-line section of the sea shore bordering his master’s land as one of the boundaries of the pasture (the goats don’t like the water and won’t try to swim away in the sea). What dimensions should the pasture be to maximize the area enclosed for the goats?
- B) A metal can with a top is to be manufactured to contain 30 cubic inches of “Tony Soprano’s Jersey Special” pizza sauce. The can will have the shape of a right circular cylinder. Find the dimensions (height and radius of the cylinder) that will *minimize the cost of the can*, assuming the metal for the top costs 1 cent per square inch, but the metal for the bottom and the curved side wall of the can costs 2 cents per square inch because of a special acid-resistant coating used to increase the shelf-life of the pizza sauce. (The inside top of the can does not need the acid-resistant coating because the cans are never completely filled with the sauce!)

- C) For some species of birds, it takes more energy to fly over water than over land (maybe because over land they can make use of updrafts). A lesser tufted grebe (this is a species of bird) leaves an island 5 km from point  $A$ , the nearest point to the island on a long straight shore. The grebe's nest is at point  $B$ , 13 km along the shore from point  $A$ . If it takes 1.4 times as much energy to fly one km over water as it does to fly one km over land, where on the shoreline should the bird head first in order to *minimize the total energy* needed for the flight from the island to the nest.
- D) Northern Iowa State Agricultural and Veterinary Junior College is building a new running track for their prize-winning track team – the “Flying Farmers.” The track is to be the perimeter of a region obtained by putting two semicircles on opposite ends of a rectangle, and that perimeter should be 440 yards in length. Due to budget problems, the administration has decided to grow sweet corn in the area enclosed by the track and sell it to a local grocery store to generate some extra revenue. Determine the dimensions to build the track in order to *maximize the area for growing corn*. (*Careful*: This is a slightly “tricky” one in that the maximum is actually at an endpoint of the interval of possible values of one of the “obvious” variables.)

### *Assignment*

Group write-ups due in class no later than Tuesday, December 8. (If you submit your solutions by the end of the class on Monday, I can return them to you in class on Tuesday before the exam.)