MATH 133 – Calculus with Fundamentals 1
Discussion Day – "Derivative Practice" – Solutions
November 5, 2015

Questions

(1) Differentiate each of these with respect to the indicated variable. Note: you will want to think first about which rule(s) you need to apply, and then apply them. *Don't worry too much about simplifying your answers – any correct form is OK for this.*

(a) $f(x) = \dfrac{x^2 + e^x}{\sqrt{x}}$

Solution: Write $\sqrt{x} = x^{1/2}$. By the quotient rule:

$$\begin{aligned} f'(x) &= \frac{x^{1/2}(2x + e^x) - (x^2 + e^x)\frac{1}{2}x^{-1/2}}{(x^{1/2})^2} \\ &= \frac{3x^2 + 2xe^x - e^x}{2x^{3/2}} \end{aligned}$$

(b) $g(t) = e^t \left(1 + \dfrac{t^2}{1 + t^2}\right)$

Solution: Use the product and quotient rules:

$$g'(t) = e^t \left(\frac{(1 + t^2)(2t) - t^2(2t)}{(1 + t^2)^2}\right) + e^t \left(1 + \frac{t^2}{1 + t^2}\right) = e^t \left(\frac{2t}{(1 + t^2)^2}\right) + e^t \left(1 + \frac{t^2}{1 + t^2}\right).$$

(c) $h(z) = \dfrac{3}{z^{2/3}} - z(e^z + 4z)$

Solution: It's easier to write the first part of this as a power instead of using the quotient rule:

$$h(z) = 3z^{-2/3} - z(e^z + 4z)$$

so

$$h'(z) = -2z^{-5/3} - z(e^z + 4) - (e^z + 4z) = -2z^{-5/3} - ze^z - e^z - 8z.$$

(2) Section 3.4 in our book builds on the way we motivated the study of derivatives by considering instantaneous velocities and slopes of tangent lines. If f is any function and $f'(a)$ exists, then we can think of $f'(a)$ as an *(instantaneous) rate of change* of f with respect to the variable in f, at a. The *units* of an instantaneous rate of change are always (units of f-values)/(units of the input variable in f). For instance, if we had a function $P(R)$ giving the electrical power (in units of watts) delivered to a device by a battery, as a function of the resistance of the device (in units of ohms), then the units of $P'(R)$ would be watts/ohm. So suppose we have a battery delivering power to a device with

$$P(R) = \frac{2.25R}{(R + .5)^2} = \frac{2.25R}{R^2 + R + .25}$$

where $R \geq 0$.

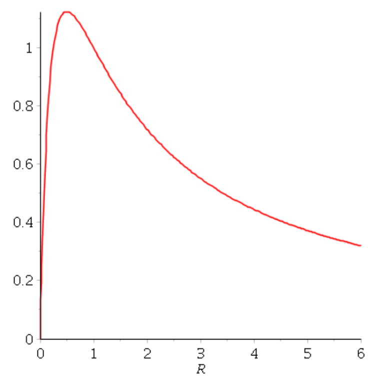

Figure 1: Plot of power P versus resistance R for $R > 0$

(a) What is the instantaneous rate of change of the power with respect to resistance when $R = 3$ ohms? (Give your answer with correct units.)

Solution: First we compute by the quotient rule:

$$P'(R) = \frac{(R^2 + R + .25)(2.25) - 2.25R(2R + 1)}{(R^2 + R + .25)^2} = \frac{-2.25R^2 + .5625}{(R^2 + R + .25)^2}.$$

We want $P'(3) = \frac{-2.25 \cdot 3^2 + .5625}{(3^2+3+.25)^2} \doteq -.13$ watts/ohm. (This indicates that the power is decreasing as the resistance increases near $R = 3$.)

(b) What is the power delivered to a device with $R = 5$ ohms? What is the instantaneous rate of change of the power with respect to resistance when $R = 5$ ohms? Give each answer with the correct units.)

Solution: The power is $P(5) \doteq .37$ watts. The rate of change of power with respect to resistance is $P'(5) \doteq -.06$ watts/ohm.

(c) Is the instantaneous rate of change of the power with respect to resistance ever equal to zero? What does that mean? Generate a sketch of the graph of $P(R)$ (R on the horizontal axis, P on the vertical axis) and show any points where $P'(R) = 0$.

Solution: Yes, the rate of change of power with respect to resistance is zero when

$$0 = P'(R) \Rightarrow -2.25R^2 - .5626 = 0 \Rightarrow R = \pm.5$$

(I'm using the simplified form of $P'(R)$ from part (a) to do this easily.) Since we only want $R > 0$, the positive root $R = .5$ is the only one. The graph of $P(R)$ for positive R above in Figure 1 indicates what is happening.