MATH 133 – Calculus with Fundamentals 1
Discussion Day on Lines and Linear Functions
September 8, 2015

Background

Every line in the plane is described by an equation of the $Ax + By + C = 0$ for some constants A, B, C. If the line is not vertical ($B \neq 0$), then it is the graph of a function. Recall that we say a function f is *linear* if $f(x) = mx + b$ for some constants m, b. The number m is called the *slope of the line* and the constant b is called the y-intercept of the line.

Questions

1) Consider lines with equations of the form $2x + cy - 3 = 0$.
   (a) For which value of c does the line contain the point $(1, 2)$?
   (b) For which value of c does the line have slope -5?
   (c) Is there any value of c such that the line is horizontal? Why or why not?
   (d) For which value of c is the line perpendicular to the line given by $5x - 3y + 1 = 0$? (Hint: What is true about slopes of perpendicular lines?)

2) Many materials, including metal rods, *expand* when they are heated. Consider a steel rod that has length L_0 (centimeters) when the temperature is T_0 degrees Celsius. If the temperature is increased by ΔT degrees Celsius, the length of the rod will change by (approximately)
$$\Delta L = \alpha L_0 \Delta T, \tag{1}$$
where $\alpha = 1.24 \times 10^{-5}$ (and ΔT is assumed to be not too large or too small). The constant α in (1) is called the *thermal expansion coefficient* of steel (the units of α are $\frac{1}{\text{degrees C}}$).
   (a) A steel rod has length $L_0 = 40$ cm at $T_0 = 40$ degrees C. What will the length be at 90 degrees C?
   (b) Find the length of the rod at $T = 50$ degrees C if the temperature at $T_0 = 100$ degrees C is 65 cm.
   (c) Express the length L as a function of T if $L_0 = 65$ cm at $T_0 = 100$ degrees C.
   (d) Explain why (1) expresses L as a linear function of T for any given L_0, T_0.

3) The volume V (in liters) of sample of 3 grams of carbon dioxide at 27 degrees Celsius was measured as a function of the pressure p (in atmospheres) with the results in the following table:

p	0.25	1.00	2.50	4.00	6.00
V	6.72	1.68	0.67	0.42	0.27

Is V (approximately) a linear function of p? Why or why not? If so, find an approximate formula $V = mp + b$. If not, can you see a equation of a different form for V as a function of p?