Mathematics 351 – Modern Algebra 1
Composition Table for S_3
August 31, 2018

Directions

S_3 is the set of all permutations of $\{1,2,3\}$. We have seen that this set of mappings is *closed under composition*. So now we want to work out the *composition table* showing the result of combining all pairs of the elements in this way. For instance,

$$\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}\circ\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}=\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}. \tag{1}$$

(As always with compositions, we are applying the mapping on the right *first*, then the one on the left!) As a head start notice that $\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}$ doesn't move any of the numbers $1,2,3$. So if we compose with that permutation on either side, the result is the same as the mapping we are composing with. Those 11 compositions are already filled in for you in the table below, as is the one from (1) above. Your job, should you choose to accept it (just kidding!), is to fill in the rest of the table. Use this convention for how to compute the composition: Top row and the left column are labeled with the six elements of S_3. The entry in row i and column j should be

$$\text{(permutation in label for row } i) \circ \text{(permutation in label for column } j).$$

Note how this works for the entry that is filled in using (1) above. Keep this table with your class notes for future reference.

$\circ$	$\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}$	$\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}$	$\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$	$\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}$	$\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}$	$\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}$
$\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}$	$\begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}$	$\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}$	$\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$	$\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}$	$\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}$	$\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}$
$\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}$	$\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}$			$\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}$		
$\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$	$\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$					
$\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}$	$\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}$					
$\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}$	$\begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}$					
$\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}$	$\begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}$					