Mathematics 351 – Modern Algebra 1
Information on Exam 2
November 6, 2018

General Information, Format and Groundrules

The second midterm exam will be given sometime around the announced date of November 16. As I mentioned in the course syllabus, we will need to decide whether you would prefer to have the exam

- as an in-class exam on Friday, November 16, *or*
- as an evening exam on Thursday, November 15 (but this conflicts with Prof. Cecil's Geometry exam), *or*
- as an evening exam on Monday, November 19.

If we decide to do an evening option, you would have 90 minutes to work on the exam, and it might be slightly longer than an in-class version. We will discuss this in class.

However we do it, this will again be a *closed book, closed notes* exam. No use of cell-phones, tablets, or other electronic devices beyond a scientific calculator will be permitted.

Topics

The exam will *make use of all the material we have discussed since the beginning of the semester, up to and including the material on the classification of finite abelian groups from Friday, November 9.* I'm saying it this way this time because, even though the problems on this exam will draw mostly on material since the first exam, all of this depends *heavily* on the earlier material too. In other words, you need to remember and use all the material on groups, cyclic groups, their structure, subgroups, cosets, Lagrange's theorem, and normal subgroups from the first exam for this exam too. The new material is outlined below.

The exam will focus partly on the key definitions and theorems we have studied. There will also be a few problems or parts of problems where you will need to apply some of those definitions and theorems to new situations.

The new topics to be covered are:

1) Factor groups G/H. Know when you can construct such a group, what the elements of the factor group are, how to compute a group table for a factor group given suitable G and H, etc. Also know how to determine the center of a group, $Z(G)$, show that it is a normal subgroup, and determine the factor group $G/Z(G)$.
2) Group homomorphisms. Know key examples like the mapping $\pi : G \to G/H$ and others we discussed in class, how to show a mapping is a group homomorphism, etc. Also know what the kernel of a homomorphism is and how to determine the kernel in explicit examples. Know the properties of group homomorphisms given in 4.12.

3) Group isomorphisms and automorphisms. Know the definitions and how to interpret the statement that groups are or are not isomorphic (thinking about the statements in Theorem 4.17).
4) First, Second, and Third Isomorphism theorems and their applications. See Problem Set 7 and the review questions below.
5) Internal direct products of subgroups and their relation to external direct products.
6) The Fundamental Theorem (Structure Theorem) for finite abelian groups. For this topic, concentrate on understanding what the theorem says and how to apply it to determine all abelian groups of a given order up to isomorphism. Do not worry about the details of the individual steps of the proof.

Review Session

If we decide to do the exam on Monday, November 19, then we can take the regular class period that morning as an optional (but recommended!) review session.

Key Theorems and Proofs to Know

1) Know how to show that the set of left cosets of a normal subgroup forms a group under the coset multiplication. This is the construction of the factor group G/H when H is normal in G from Theorem 4.6 in Lee.
2) Know the proofs of both parts of Theorem 4.11 in Lee.
3) Know the proof that every group of order $2p$ (p an odd prime) is isomorphic to either $\mathbf{Z}_{2p}$ or D_{2p} (Theorem 4.15).
4) Know the proof of the First Isomorphism Theorem (Theorem 4.18).

Suggested Review Problems

From Lee:
4.12, 4.13, 4.17, 4.19;
4.21, 4.23, 4.27, 4.29;
4.31, 4.33, 4.35, 4.37;
4.41, 4.45;
4.51, 4.53;
5.11, 5.13, 5.15;
5.21, 5.22.