Mathematics 351 – Modern Algebra 1
Information on Exam 1
September 25, 2018

*General Information, Format and Groundrules*

The first midterm exam will be given during the week of October 1. As I mentioned in the course syllabus, we will need to decide whether you would prefer to have the exam

- as an in-class exam on Friday, October 5, *or*
- as an evening exam on Thursday, October 4.

If we decide to do the evening option, you would have 90 minutes to work on the exam, and it might be slightly longer than an in-class version. We will discuss this in class on Wednesday, September 26 and decide which format you would prefer.

*Either way*, this will be an *closed book, closed notes* exam. No use of cellphones, tablets, or other electronic devices beyond a scientific calculator will be permitted.

*Topics*

The exam will cover the material we have discussed since the beginning of the semester, up to and including the material on normal subgroups on Wednesday, September 26. This is the material from sections 2.4 and 2.5, all of Chapter 3, and section 4.1 in Lee.

The exam will focus partly on the key definitions and theorems we have studied. There will also be a few problems or parts of problems where you will need to apply some of those definitions and theorems to new situations.

The topics to be covered are:

1) The definition of a group and key examples such as $\mathbf{Z}$, $\mathbf{Z}_n$, $S_3$, groups of matrices, cyclic groups, $D_8$
2) Basic properties of groups and group tables
3) Powers and orders of elements in a group
4) Subgroups, especially the cyclic subgroups of given groups generated by given elements of the group
5) Cyclic groups and their properties (see list of theorems to know below); for finite cyclic groups, know how to determine the generators and elements of other orders, know how to determine how many of them there are via the Euler $\phi$-function
6) Cosets and Lagrange's theorem, plus consequences
7) Normal subgroups

*Review Session*

We will review for the exam in class on Wednesday, October 3. (The course schedule online has been revised to account for this.) Of course this means you will not have much time between then and the exam to do further preparation. *Begin your studying before the review session, please!*

*Key Theorems and Proofs to Know*

1) Be prepared to decide whether a given set with a given operation is or is not a group using Definition 3.1 in Lee. Similarly, be prepared to show that a given subset of a group is or is not a subgroup using either Theorem 3.10 in Lee or the streamlined version we discussed in class.
2) Know how to show that if $a$ is an element of order $n$ in a group, then $a^i = e$ if and only if $i \equiv 0 \bmod n$, and $|a^i| = n/\gcd(n, i)$ (see class notes and Corollary 3.2 in Lee).
3) Know how to prove that every subgroup of a cyclic group is cyclic (Theorem 3.16) and that if $G$ is finite cyclic of order $n$, then there is exactly one subgroup of $G$ of order $m$ for each $m$ satisfying $m|n$ (Corollary 3.3).
4) Know the statement and proof of Lagrange's Theorem, taking facts about equivalence classes as given. The full reasoning for the proof is spread over Lemmas 3.1 and 3.2, plus Theorems 3.20 and 3.21 in Lee. We also split things up that way in class, but you should be able to do the whole proof start to finish.
5) Be prepared to decide whether a given subgroup $H$ of a group $G$ is or is not a normal subgroup of $G$ using one of the equivalent forms given in Theorem 4.3 in Lee.

*Suggested Review Problems*

From Lee:
2.37, 2.39, 2.40;
3.1, 3.3, 3.5;
3.7, 3.9, 3.13, 3.15;
3.17, 3.21;
3.23, 3.25, 3.27, 3.29;
3.33, 3.36, 3.37, 3.39, 3.41, 3.42;
3.43, 3.45, 3.47, 3.51;
3.53, 3.55, 3.57.
4.1, 4.5.