

MATH 110-2 – Algebra Through History
Viète’s Take on Diophantos II, 8
November 20, 2019

Background

To make sure we understand what Viète is saying here, let’s work out a couple of examples. (Note: This is also related to one of the problems on this week’s problem set!)

Questions

Suppose we want to start with $F = 39$ and write $F^2 = 1521$ as a sum of two other squares.

A. Probably the most convenient right triangle to use is the $(5, 12, 13)$ -right triangle, or $B = 5$, $D = 12$, $Z = 13$ (note that $B^2 + D^2 = 25 + 144 = 169 = 13^2$, so this is a valid “Pythagorean triple” to use).

1. What is the ratio $\frac{F}{Z}$ here?

2. What are $X = \frac{BF}{Z}$ and $Y = \frac{DF}{Z}$? Check that $X^2 + Y^2 = F^2$.

B. Repeat the calculations in part A if you started from the $(3, 4, 5)$ -right triangle instead, or $B = 3$, $D = 4$, $Z = 5$. (Why did I say that the choice in A was the most convenient?)

C. Thinking about A and B together and generalizing, how many different ways are there to write F^2 as a sum of two squares (of rational numbers)? Does it seem clear that Viète is aware of what you said? Did it seem as though Diophantos was?