MATH 110-02 – Algebra Through History
The Tartaglia-Cardano Formula for Cubics
November 11, 2019

Background

We just saw that the formula Tartaglia found and Cardano printed in his *Ars Magna* can be written as follows. For the "cubes and roots equals numbers" case $x^3 + px = q$, (one) solution is

$$x = \sqrt[3]{\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$$

Tartaglia and Cardano would have been considering the case $p, q > 0$ but we can see that the formula is actually valid when $p < 0$ or $q < 0$ as well(!)

Questions

A. Use the formula to solve

$$x^3 + 3x = 36$$

with $p = 3$ and $q = 36$. Note: There is an "obvious" real solution $x = 3$ here. Is it clear your answer is giving that same value??

B. What happens if you use the formula to solve:

$$x^3 - 6x - 2 = 0$$

where $p = -6$ and $q = 2$? (There are actually 3 distinct real roots as you can see by plotting $y = x^3 - 6x - 2$ on a graphing calculator.)