MATH 110-02 – Algebra Through History
Sample Midterm Exam Questions

Disclaimer: The following questions indicate the rough length of the coming midterm and the "mix" of different kinds of questions I might ask. The actual exam might be set up somewhat differently and it might draw on other topics from the review sheet.

I. Refer to the Old Babylonian tablet YBC 7289 in Fig 1 (at top of next page).

A) The base-60 form of the number written directly over the diagonal of the square has just one base-60 digit to the left of the semicolon separating the whole number part from the fractional part. Give all the base-60 digits, separated by the semicolon and commas.

B) Express the whole number from part A in base 10. (It's OK to leave this as a sum of fractions if you don't have a calculator.)

II.

A) What does it mean to say that that the diagonal of a square and the side of that square are *incommensurable*?

B) Give the proof the ancient Greeks found for the fact in part A.

C) What consequences did the discovery of incommensurable magnitudes have for the way the Greeks thought of (pure) mathematics?

III. Short answer. Answer any four of the following. If you answer more than four, the best four will be used.

A) What is the main feature that makes the presentation of the number theoretic problems in Diophantus' *Arithmetica* different from earlier mathematics and a forerunner of what we do now.

B) What is the "algebraic form" of the fact proved in Proposition II.5 in Euclid's *Elements*? (The statement: If a straight line is cut into equal and unequal pieces then the rectangle contained by the unequal pieces of the whole line, plus the square on the difference between the equal and unequal pieces is equal to the square on half of the line.)

C) What geometric fact did Archimedes have inscribed on his tombstone?

D) What are the approximate dates of the YBC 6967 and YBC 7289 tablets? About when did Euclid live?

E) What does the $M^o 20 ς 1$ in Diophantus' Proposition I.4 mean?

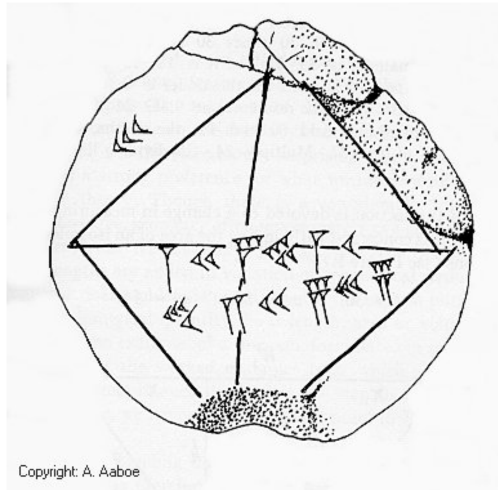

Figure 1: Drawing of the YBC 7289 tablet by Asger Aaboe. I do not have his permission to use this, but he died in 2007, so I think the copyright issue is effectively moot(!)

G) What is "Fermat's Last Theorem" and what is its connection with Diophantos? When and by whom was this finally solved?

H) How would the Old Babylonian scribes have approached a large calculation such as multiplying $23, 41; 16$ by $17, 28; 30$? (You don't need to do the calculation.) Compare this with what we need to know to do multiplication in our base-10 positional number system.

IV. Essay. You have the choice of responding to *either prompt 1 or 2*. State which one you have chosen at the start of your essay.

1) A certain older history of mathematics says, flatly, that "the distinguishing feature of Babylonian mathematics is its algebraic character." Of the historians we have mentioned, who would agree with this claim and who would disagree? Explain using the the interpretations your historians would give for the YBC 6967 problem of (what we would phrase as) solving the equation $x = 60/x + 7$.

2) George G. Joseph, the author of another book on the non-European roots of modern mathematics called *The Crest of the Peacock*, offers this overall evaluation of the ultimate impact of Greek geometry: "There is no denying that the Greek approach to mathematics produced remarkable results, but it also *hampered* the subsequent development of the subject. ... Great minds such as Pythagoras, Euclid, and Apollonius spent much of their time creating what were essentially abstract idealized constructs; how they arrived at a conclusion was in some way more important than any practical significance." First, what does the last sentence mean? Would this criticism seem to be apt for Diophantos' *Arithmetica* as well? Is it necessary for all the mathematics we learn and do to have practical usefulness or significance?