Analysis and synthesis in ancient geometry

Pappus of Alexandria, *Mathematical Collection* (ca. 340 CE), VII:1–3

The so-called Treasury of Analysis is, to put it shortly, a special body of doctrine provided for the use of those who, after finishing the ordinary *Elements*, are desirous of acquiring the power of solving problems which may be set them involving [the construction of] lines, and it is useful for this alone. It is the work of three men, Euclid, the author of the *Elements*, Apollonius of Perga, and Aristaeus the elder, and proceeds by way of analysis and synthesis.

Analysis, then, takes that which is sought as if it were admitted and passes from it through its successive consequences to something which is admitted as the result of synthesis: for in analysis we admit that which is sought as if it were already done and we inquire what it is from which this results, and again what is the antecedent cause of the latter, and so on, until by so retracing our steps we come upon something already known or belonging to the class of first principles, and such a method we call analysis as being solution backwards.

But in *synthesis*, reversing the process, we take as already done that which was last arrived at in the analysis and, by arranging in their natural order as consequences what before were antecedents, and successively connecting them one with another, we arrive finally at the construction of what was sought; and this we call synthesis.

Now analysis is of two kinds, the one directed to searching for the truth and called *theoretical*, the other to finding what we are told to find and called *problematical*. (1) In the *theoretical* kind we assume what is sought as if it were existent and true, after which we pass through its successive consequences, as if they too were true and established by virtue of our hypothesis, to something admitted: then (a) if that something admitted is true, that which is sought will also be true and the proof will correspond in the reverse order to the analysis, but (b) if we come upon something admittedly false, that which is sought will also be false. (2) In the *problematical* kind, we assume that which is propounded as if it were known, after which we pass through its successive consequences, taking them as true, up to something admitted: if then (a) what is admitted is possible and obtainable, that is, what mathematicians call given, what was originally proposed will also be possible, and the proof will again correspond in the reverse order to the analysis, but if (b) we come upon something admittedly impossible, the problem will also be impossible.