MATH 110–2 Algebra Through History
Information on Research Papers
October 15, 2019

Assignment

As you know from the course syllabus and the schedule posted on the course homepage, one of the final assignments for our course will be a medium-length research paper on a topic related to our historical survey of algebra. You should aim for about 6-8 pages in length (slightly longer or slightly shorter that this is also OK). As with any research paper of this kind, you should identify the sources of information that you include that does not come from your own work (e.g. any direct quotations, any mathematical examples you find in your sources, ideas about interpreting the original sources, etc.). You may do this with footnotes or endnotes as you prefer. You should *also* include a listing of all sources: books, articles, web pages, etc. that you consult on the final page of your paper. I'm not picky about what form you use for the citations in your listing, but please pick one of the standard ones (e.g. MLA, APA, etc.) and follow it consistently.

The timetable for the assignment is as follows:

- The paper "proposal:" by 5:00pm on November 15 (at the latest), inform me which of the topics indicated below you would like to write on. *This is especially important if you want to design a topic of your own; you will need to get my approval in that case.* In several cases where there are a number of different directions a paper could take, I'll also ask you to give me a preliminary description of which aspects you would like to focus on at this time. If there are any issues with what you are proposing, I will get back to you early the following week and we can discuss them in office hours.
- The final papers will be due no later than 5:00pm on December 13 (the last day of class). These will be submitted as Google docs or Word documents to:

jlittle@holycross.edu.

Early submission is also a possibility and that will gain the undying gratitude of your reader :)

Please get started thinking about which topic you would like to address early. I don't have any problem if more than one of you want to write on a particular topic, but we should try to spread out as much as possible so that there will not be too much demand for any particular book as a source. This will be especially important if you need to use interlibrary loan to get sources not contained in our library's collection. *Ms. Merolli, our Science Librarian, will be more than happy to help out with searches and obtaining sources if you need assistance.* I can also help you track things down in many cases, and I will be happy to make photocopies of sections of books in my personal collection if you want to consult them.

Some ideas about paper topics

1. *Ancient Egyptian Mathematics.* At almost the same time as the Old Babylonian period in Mesopotamia, scribes in Middle Kingdom Egypt were considering many quite algebraic questions. For this paper topic, you would research how the Egyptians dealt with multiplication and division of numbers and how they worked with fractions. Then you would discuss a selection of problems from the principal surviving mathematical texts of this period: the *Rhind and Moscow papyri.* See George Joseph's book "The Crest of the Peacock" for a good discussion of this. There is also a brief discussion of Egyptian mathematics at the start of Chapter 1 in Katz and Parshall, but they don't think of what the Egyptians did as very "algebraic;" others might disagree(!) Katz's sourcebook *The Mathematics of Egypt, Mesopotamia, China, India, and Islam* contains other examples.

2. *More on Old Babylonian Mathematics.* There is quite a bit more known about both the sorts of problems that were studied in Old Babylonian mathematics and the social context of the scribal schools where the tablets we discussed in class were created than we had time to cover in class. There are several directions that one might take here. For instance,
   (a) The book *Mathematics in Ancient Iraq* by Eleanor Robson is an excellent discussion of the social context and what we know about the scribal schools.
   (b) The articles *Neither Sherlock Holmes nor Babylon: A Reassessment of Plimpton 322*, Historia Mathematica 28 (2001), 167–206, and *Words and Pictures: New Light on Plimpton 322*, American Mathematical Monthly, February 2002, 105–120, both by Eleanor Robson, discuss another very famous Old Babylonian tablet that we mentioned briefly, and various modern attempts to account for the mathematics it contains. She also discusses the pitfalls of treating ancient texts as "mathematical puzzles" to be solved with modern ideas and techniques without taking account of what the creators could have been thinking.
   (c) There are many other Old Babylonian mathematical tablets that have been deciphered. Victor Katz's sourcebook *The Mathematics of Egypt, Mesopotamia, China, India, and Islam*, and Jöran Friberg's *A Remarkable Collection of Babylonian Mathematical Texts* would be good places to consult to get an idea of the range of different things they did.

3. *Algebraic Thought in India.* We did not discuss anything much about mathematics from the subcontinent of India, but there is a rich tradition with much algebraic content there. Moreover, the interconnections between India, the Middle East, and Europe are only now becoming better understood. Some good sources to consult here are Chapter 6 of *Taming the Unknown*, Victor Katz's sourcebook *The Mathematics of Egypt, Mesopotamia, China, India, and Islam*, and George G. Joseph's *The Crest of the Peacock.* There are several ways a paper here might develop:
   (a) A survey of algebraic ideas in Indian mathematics
   (b) A more in-depth treatment of a major figure like Brahmagupta or Bhaskara II, or Arhyabata.

(c) For students with a bit more mathematical background, a very interesting topic is the question to what extent work on infinite series in medieval Kerala (a state in south India) anticipated European work in calculus. Katz and Parshall (in footnote 44 on page 131) mention that there were Jesuit missionaries in India by the 17th century, and they say that "there is no evidence from that time of the transmission of and actual transplantation of western mathematical ideas there." In fact, other writers such as George G. Joseph in another book of his called *A Passage to Infinity* have asked whether the influence might actually have *gone the other way*: Might this Indian mathematics have made it back to Europe by way of those very Jesuit missionaries and stimulated the development of European mathematics?

4. *Archimedes' Quadrature of the Parabola.* (This topic is probably the most mathematically sophisticated of the lot – only recommended for those who are willing to "dig deep" into a piece of amazing mathematics.) In the work mentioned above, Archimedes proves that the area of a segment of a parabola (the region between the parabola and one of its chords) is equal to 4/3 of the area of the triangle formed by the endpoints of the chord and a third point on the parabola called the *vertex* of the segment. (The vertex is the point where the tangent line to the parabola is parallel to the chord; it also happens to be point with the same x-coordinate as the midpoint of the chord, if the parabola is $y = x^2$ in a coordinate equation.) For this topic, you could work through either one of the two arguments that Archimedes gives for this. The first involves an ingenious method of slicing the segment up into infinitesimally thin strips and "weighing it on a balance." The second finds the area by filling up the region between the triangle and the parabola with other triangles, then adding their areas with what turns out to be a *geometric series*. (Note: Archimedes gives the second proof because he says he knows that there are some questionable aspects of the first one – it's far ahead of its time, in fact, and anticipates ideas from the integral calculus that were not formally developed until about 1800 years after Archimedes' time.)

5. *Algebraic Thought in China.* We did not discuss anything much about mathematics from China, but there is a very rich tradition with much algebraic content there. Moreover, the interconnections between China, India, the Middle East, and Europe are only now becoming better understood. Some good sources to consult here are Chapter 5 of *Taming the Unknown*, Victor Katz's sourcebook *The Mathematics of Egypt, Mesopotamia, China, India, and Islam*, and George G. Joseph's *The Crest of the Peacock*. There are several ways a paper here might develop:
   (a) A survey of algebraic ideas in Chinese mathematics
   (b) A more detailed look at the Chinese Remainder Theorem and applications
   (c) A more detailed look at Chinese methods for solving simultaneous systems of linear equations. What they did there turns out to be very close to the later matrix methods for these problems developed in Europe in the 1800's(!)

6. *Islamic Mathematics and the Translation Movement in 8th - 10th Century Baghdad.*

Victor Katz's sourcebook *The Mathematics of Egypt, Mesopotamia, China, India, and Islam* contains a number of translations of original works from this and later periods.

(a) The methods developed by Omar Khayyam for solving cubic equations would be an excellent topic.

(b) An interesting question is just why the translation of Greek scientific and mathematical texts assumed such a large importance during the Abbasid period in Baghdad. A related question is: what was the nature and the role of the institution known as the *bayt al-hikmah* (House of Wisdom)? The book *Greek Thought, Arabic Culture* by Dmitri Gutas is one of the most balanced and careful examinations of this question. There are a number of other treatments as well, but modern tensions between the West and the Islamic world have tended to influence scholarship on these issues too, with Western authors sometimes discounting the importance of the work done in Baghdad, and Islamic authors claiming too much importance for it(!)

7. *Mathematical Lives.* If you want to work on a "mathematical biography" of an important figure, the following would be interesting choices. Try to bring out especially the connections with the history that we have discussed.
   (a) Hypatia (and the end of the Alexandrian mathematical tradition)
   (b) Leonardo Pisano (also known as "Fibonacci")
   (c) Luca Pacioli (also known as the father of double-entry bookkeeping!)
   (d) Girolamo Cardano
   (e) Niccolo Tartaglia
   (f) Ludovico Ferrari (and quartic equations)
   (g) Francois Viète (Vieta)
   (h) Évariste Galois (and the problem of solution of general polynomial equations "by radicals")

8. *A topic of your own.* If there is some topic that has occurred to you that you don't see above, please put together a reasonably detailed description of what you would like to write on and consult with me to get my approval before you begin to work in earnest.