

MATH 110 – Algebra Through History
Problem Set 3, **due:** 5:00pm on Wednesday, November 6.
Some problems from Al-Khwarizmi

I. Solve the following quadratic equations using the methods described in Chapters I through VI of “Book of Algebra and Almucabola”

- A. Six squares equal 486 units. Find the root. (See Chapter II.)
- B. A square and 8 roots equal 20 units. Find the root. (See Chapter IV.)
- C. A square and 48 units equal 14 roots. Find the root. (See Chapter V.)

II. Why does Al-Khwarizmi treat problems like the ones in B and C from question I as different types? Explain using modern algebraic notation.

III. Illustrate how the geometric reasoning Al-Khwarizmi gives for the “squares and roots equals numbers” case applies to the problem from I. B. (See pages 77 and 79.)

IV. Here’s a translation of one of the inheritance problems from Al-Khwarizmi’s book: A man dies, leaving two sons behind him, and bequeathing one-third of his capital to a stranger. He leaves ten dihrms of property and a claim of 10 dirhams upon one of the sons. [Comment: That is, one of the sons owes the father 10 dihrms. The question is to determine how much of the debt the son who owes the 10 dihrms must pay to the estate so that everyone would come out “even” according to the terms of the inheritance when everything was distributed. Because of the size of the debt you can see that that one son is not going to get anything back – he’ll pay something less than 10 dihrms and he’ll be “even” – the amount he pays in will be the same as the amount he would receive from the estate if the transactions were done in two stages – that is, if he paid first and then the estate was distributed. The purpose of the calculation is to do the transaction all in one step.]

Computation: You call the sum which is taken out of the debt *thing*. [Comment: This means that the son who owes the 10 dihrms only has to pay *thing* out of that debt.] Add this to the capital, which is 10 dihrms. The sum is ten and *thing*. Subtract one-third since he has bequeathed one-third of his property, that is, three dihrms and one-third, plus one-third of *thing*. The remainder is six dihrms and two-thirds plus two-thirds of *thing*. Divide this between the two sons. The portion of each of them is three dihrms and one-third, plus one-third of *thing*. This is equal to the *thing* which was sought for. Reduce it, by removing one-third from *thing*, on account of the other third of *thing*. There remain two-thirds of *thing*, equal to three dihrms and one-third. It is then only required that you complete the *thing*, by adding to it as much as one half of the same; accordingly you add three and one-third as much as one half of them. This gives five dihrms, which is the *thing* that is taken out of the debt.

Explain the logic behind this solution, using modern algebra as appropriate.