MATH 110-02 – Algebra Through History
Problem Set 2, **due:** 5:00pm on Monday, October 7.
Diophantus, the *Arithmetica*

Here is a more-or-less literal translation of Diophantus' Proposition 12 from Book I:

Book I, Proposition 12. To divide a given number into two parts *twice*, so that one of the numbers in the first decomposition has a given ratio to one of the numbers in the second decomposition, and the other number in the first decomposition has another given ratio to the other number in the second decomposition.

Let it be proposed to divide 100 into two parts twice, so that the larger number in the first decomposition is in 2^{pl} ratio with the smaller number in the second decomposition and the larger number in the second decomposition is in 3^{pl} ratio with the smaller number in the first decomposition.

Let the smaller number in the second decomposition be set out as $\varsigma 1$ then the larger number in the first decomposition will be $\varsigma 2$. The smaller number in the first subdivision is then $M^o 100 \Lambda \varsigma 2$. And since the triple of this number is the larger number in the second subdivision, that number is $M^o 300 \Lambda \varsigma 6$. But the sum of $\varsigma 1$ and $M^o 300 \Lambda \varsigma 6$ is 100. So $M^o 100$ is $M^o 300 \Lambda \varsigma 5$. Therefore ς is 40.

To what was set down.

- The larger number in the first decomposition is $\varsigma 2$, which is $M^o 80$.
- The smaller number in the first decomposition is $M^o 100 \Lambda \varsigma 2$ which is $M^o 20$.
- The triple of the smaller number in the first decomposition is $M^o 60$ (This is the larger number in the second decomposition.)
- The smaller number in the second decomposition is ς, which is $M^o 40$.

And the verification is evident.

A) Translate Diophantus' proof into modern algebra and show that his method is correct (for his numbers).

B) Show how the method given here could be generalized if the given number is any n, the two decompositions are $n = x_1 + x_2$ and $n = x_3 + x_4$ with $x_1 > x_2$ and $x_3 > x_4$ and we are given the ratios $x_1 = ax_4$ and $x_3 = bx_2$ with a, b some positive integers.

C) ("Thought question") Do you think that Diophantus' "proof" counts as a general proof for solving the type of problem he is describing? Does it matter that he is showing how to solve the problem for particular values of the numbers?