MATH 110 – Algebra Through History
Problem Set 1 – Babylonian Mathematics
due: Friday September 20, by *5:00pm*

I. A Old Babylonian tablet from about 1700 B.C.E (now held by the Louvre in Paris) has the following problem: "Find how long it will take a certain sum of money to double itself at compound annual interest of 20%." This means that you will have $1.2\times$ the original amount after 1 year, $(1.2)^2\times$ the original amount after 2 years, $(1.2)^3\times$ the original amount after 3 years, and so on. The question is: How many years will be needed until you have twice the original amount? (Fractional parts of years are also allowed.)

A) The Babylonian method of solution (written with base 10 numbers and in modern language, of course) was this: First compute the powers to see that $(1.2)^3 = 1.728$ and $(1.2)^4 = 2.0736$. So the doubling will happen between the 3rd and 4th year. To find the doubling time, find the point on the straight line through $(3, (1.2)^3) = (3, 1.728)$ and $(4, (1.2)^4) = (4, 2.0736)$ with $y = 2$. The x-coordinate of that point is the doubling time. *Note: This is true approximately, but not exactly.* Carry out the calculations to find this time.

B) The Babylonian tablet gives the answer by this method as the base 60 number

$$3; 47, 13, 20$$

(with fractional part). Is this correct (does it agree with with you did in part A)?

C) Here is a modern solution of this problem, giving the exact value of the doubling time using the concept of *logarithms* (which were only invented in the 1600's CE): From the equation $2 = (1.2)^x$ take the natural log of both sides to yield $\ln(2) = x\ln(1.2)$ so $x = \frac{\ln(2)}{\ln(1.2)}$. The ratio $\frac{\log_{10}(2)}{\log_{10}(1.2)}$ gives the same result if you are more comfortable with common logarithms. Using a calculator, compute a decimal approximation and compare this value to the Babylonian result from part B. How close were they?

II. A number of Old Babylonian tablets containing values of $n^3 + n^2$ for $n = 1, ..., 30$ have been found.

A) Make such a table for $n = 1, \ldots, 10$.

B) Use it to solve the cubic equation $x^3 + 2x^2 = 3136$. (Note: The trick is to multiply both sides of the equation by an appropriate number A chosen so that $Ax^3 = n^3$ and $2Ax^2 = n^2$ are both true for some integer n. The Babylonian scribes would be finding this A by trial and error. You can use modern algebra to make the problem easier!) *Answer:* $x = 14$*, but full details of how this is derived must be shown for credit.*

C) A tablet of about 1800 B.C.E. from Susa in present-day Iran asks for a solution of the system of equations

$$\begin{aligned} xyz + xy &= 7/6 \\ y &= 2x/3 \\ z &= 12x \end{aligned}$$

Use the last two equations to eliminate y, z and get an equation in x alone. Then use your table from part A) to find the solution.