MATH 110 - 02 – Algebra Through History
Information for Final Exam
December 11, 2019

General Information

The final exam will be given in our regular classroom at *11:00am on Wednesday, December 18*, as announced by the Registrar. It will cover the historical material we have studied since the start of the semester.

This exam will be comprised of about 60% on short answer questions related to the history we have discussed, and 40% on a roughly 2-page essay on an assigned topic (see below). If you are well-prepared, I'm expecting the exam will take about 1.5 hours to complete, but you will have the full 2.5 hour exam period to work on it if you need more time. *Note:* I will *not* be asking any questions where you need to solve mathematical problems, although you will see that some of the historical questions will ask for details about the mathematical contributions of the people we studied.

Mathematical/Historical Topics

- From our study of Old Babylonian mathematics:
  a. Know the approximate historical period represented by the Old Babylonian mathematical texts.
  b. Know the base-60 number system and number symbols and be able to read "the real thing" and convert to base-10 form (given information about where the fractional part of the number starts).
  c. Know and understand what the tablets YBC 6967 and YBC 7289 contain
  d. The role of addition, multiplication, reciprocal, $n^3 + n$, ... tables in Babylonian arithmetic
  e. Know the different interpretations of the Babylonian "quadratic algebra" problem texts like YBC 6967 given by Otto Neugebauer and more recent historians like Jens Hoyrup and Eleanor Robson.
- From our study of the Greek algebra and number theory:
  a. Know the approximate historical periods of Euclid and Diophantus.
  b. The meaning of incommensurability of magnitudes, the discovery of incommensurable magnitudes like $\sqrt{2}$, know the way that the diagonal of a square was proved to be incommensurable with the side of the square (see class materials for September 27 for the proof from "Book X, Proposition 117" in Euclid). Know the way the Greeks dealt with the fact that these incommensurable magnitudes exist.
  b. Euclid's "geometric algebra" from Book II of the *Elements*; Know the proof of Proposition 5 (the geometric form of the identity $ab + ((a + b)/2 - b)^2 = ((a + b)/2)^2$.) See pages 7 and 8 of the class materials for September 25.
  c. Diophantus' *Arithmetica*. In particular, know the algebraic symbolism Diophantus introduced and be able to "decode" the proof/example for Proposition 4 from

Book I (translate his notation into modern algebra, carry out his steps and solve the problem).

d. How know Proposition 8 from Book II of the *Arithmetica* led to "Fermat's Last Theorem" and when by whom that was finally established.

e. Know the time frame for Pappus of Alexandria, his mathematical contributions, and in general terms what his ideas about *analysis* and *synthesis* in mathematics were.

- From our study of medieval Islamic mathematics:

  a. Know when and where Al-Khwarizmi was active, what his main contributions to algebra were, how he presented solutions of different types of quadratic equations, and in particular how he said to solve the "squares equal to roots and numbers" case.

  b. Know the other Islamic mathematicians we discussed (Thabit Ibn Qurra, Umar al-Khayyami = Omar Khayyam), and their contributions to algebra that we discussed

  c. Know (in general terms) how the Greek mathematical works translated into Arabic and the works of the Islamic mathematicians were transmitted to western Europe after about 1100 CE. Know the roles of Gerbert of Aurillac, Gerard of Cremona, Robert of Chester, and Leonardo Pisano (also known as Fibonacci).

- Renaissance algebra:

  a. Tartaglia, Ferrari, Cardano, and the solution of cubic and quartic equations. Know the period these mathematicians lived and the story of how the solution of these equations became known. Know Tartaglia's formula for solving cubics of the "cubes and roots equals numbers" form, i.e. $x^3 + px = q$. Recall that this is:

$$x = \sqrt[3]{\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$$

  b. François Viète and *The Analytical Art.* Know the timeframe, the major characteristics of what he did–for instance, how did his work go beyond what Diophantos had done before? Also know Viète's point of view about Al-Khwarizmi and the Islamic algebra he had inherited, and how he based his work on ideas from Pappus of Alexandria (see above). Know Viète's Homogeneity Principle and how that factored into his ideas about algebra.

  c. Descartes, *La Géometrie* and the development of "analytic geometry."

Essay

The essay question will be the following:

Although this has mainly been a course about the historical development of algebra, when you look back at what we have learned, you should see that one consistent theme throughout has been the ways algebra and geometry have been connected with each other, and the changing ways mathematicians have thought about that relationship. Discuss the relationship between algebra and geometry in each of the following periods, or mathematical works.

(1) The Old Babylonian period and the problem texts such as YBC 6967. And is there a difference of opinion about how to interpret what is algebra and what is geometry there?
(2) The presentation of elementary mathematics in Euclid's *Elements*, especially in Book II.
(3) The *Arithmetica* of Diophantos.
(4) The *Hisab al-jabr w'al-muqabala* of Al-Khwarizmi.
(5) *La Géometrie* of René Descartes.

[Note: This next part is asking for an opinion; no single "right answer":] In teaching algebra today, which way of dealing with algebra and geometry do you think would help students most to learn the subject? And would knowing some of this history also help?

Miscellaneous Groundrules

No use of cell phones, pagers, I-pods, or any other electronic devices beyond a calculator will be allowed during the exam – turn them off and stow them in your backpack.