MONT 107Q – Thinking About Mathematics
Final Projects
March 6, 2017

General Information

As announced in the course syllabus, one of the assignments for the seminar this semester will be a final project. You will be working on this project in *teams of 2* and the goals will be to prepare a roughly 15-page research paper and an oral presentation of approximately 25 minutes to the class on your project.

Schedule and Deadlines

- **On or before Friday, March 17** – Inform me *by email* which general topic you want to work on and with whom you will be working. Since each project area has several different directions that might be pursued (see the descriptions below), write up a paragraph giving a description of the aspect(s) you would like to work on. If you need assistance in forming "teams," I will be happy to help with that. Ideally, each group will work on a different topic, although as noted below the areas are large enough that if more than one group wants to try that, there will be ways to "split up" the topic into several parts. We can discuss the possibilities before this is due if you would like.
- **Friday, April 7** – Each group will submit, *by email*, a bibliography of the sources to be used for your project. You should identify *at least six books, articles, or web sites, including no more than two web sites*, that will be relevant. For each of your sources, write up a short paragraph giving a rough description of how that source relates to your main topic, what kind of information you will take from it, and how you will be using it (including a preliminary estimate of how reliable you think the information there is). The project descriptions below contain some first places to look, but you should plan to spend some time searching for additional sources. This annotated bibliography will count for 10% of the final project grade.
- **Between April 10 and April 21** – Each team will meet with me during office hours (or at another time if that is not convenient) for a progress report and a chance to discuss any questions that have come up as you have started to work on the project. I will be happy to meet to talk over any aspect of the project at other times too, of course.
- The presentations will be given in the final meetings of the seminar – May 1, 3, 5, and 8. We will have two presentations each of those days; the exact scheduling will be determined later.
- The oral presentation will receive a separate grade, and will count for 30% of the total on the final project.
- **Monday, May 8** – All final project papers will be due (via email) by 5:00pm. The grade for the paper component will account for 60% of the total on the final project. This assignment will take the place of a final examination for this course.

Other Information

- See the day-by-day course schedule for the other assignments for the course so that you can plan your work for the project.
- Ms. Merolli, our Science Librarian, will be visiting our class during the week of March 20 to introduce herself and give some introductory information about using the library resources to identify sources for your project. She will be more than happy to assist you in the important process of assembling the resources you will use.
- The presentations can be done with PowerPoint, or without visuals, as you prefer.
- I will ask each group to do a "dry run" of their presentation with me at least one day before you go in front of the class. The purpose of this is to give you some feedback about what is working and what is not, and to give you some practice to minimize the effect of "nerves" when the time comes for the real thing.

The Final Project Paper

This writing assignment will be different from the other writing assignments on *All the Pretty Horses* and the *Metamorphoses*. The goals of this assignment are for you to collect information about your topic from the various sources you find, and then present your analysis of that information. The evaluation of your project reports will be based on how well you have addressed the following guidelines and expectations:

- Distill your investigations into a central argument. A good research paper of this kind should be more than just a compilation of information from all the different sources you consulted. It should clearly show that you have thought independently about the information you found, that you have weighed the evidence for the various claims that were made in your sources, and that you have a central theme or argument about your topic that you want to present. It is certainly permissible to say you disagree with points of view presented in some sources, if you can explain why you think that and back up your opinions with appropriate evidence.
- The paper should be *well organized* and the writing should give the reader a clear indication where you are heading with your central argument at all times.
- Pay special attention to the first few paragraphs that will serve as an introduction. Catch the reader's attention, explain the significance of the topic or theme you will discuss. Say what you will do in general terms, without going into all the details from the start.
- Also pay special attention to the final few paragraphs, which will serve as a conclusion for your paper. Don't overstate the importance of your findings, and be honest if there are limitations. You might discuss how your investigations could be continued in further research.
- Give proper credit to sources you consulted that contributed to your ideas about the questions you studied. (In a longer thesis, it would be expected that another section reviewing the most relevant contributions of previous work on related subjects would be included – that kind of full literature review is *not expected for this assignment.*) Use footnotes or endnotes to identify direct quotations from your sources, and also to indicate the sources that contribute to specific points you are making.

- In a *References* section at the end, include all books, articles, websites you used in the preparation of the work. For books, give the author(s), title, publisher, place and year of publication. For articles, give the author(s), title, journal name, volume, year, and pages. For any websites, give the full URL, the author (if that can be determined), and the date you consulted.
- Be clear, concise, and correct in your writing. Aim for *no typos, misspellings, or grammatical problems*. But even more importantly, each paragraph should have a clearly evident purpose in relation to your main argument.
- Use figures, graphs, etc. sparingly in the main text. (If you want to include more of these, that can be done in an additional Appendix section at the end.)
- Proofread your work carefully and have an "impartial" reader or readers look at it and give you comments. This can be one of the other teams or me. Be prepared and willing to *revise* your work based on the comments you get. Of course, this means that the writing must not be put off until the afternoon of May 8(!) Be sure you get started early enough so that the input can be put to productive use.

Project Ideas

Area 1 – Jesuit Mathematicians, Astronomers, and Missionaries

A number of Jesuit mathematicians, astronomers, and missionaries have been involved in various ways in mathematical and historical topics we have touched on this year.

For instance, Christopher Clavius, S.J. was a geometer and astronomer from the early years of the Society of Jesus who played a major role in mathematics education in Europe. He was instrumental in the inclusion of mathematics in the curriculum of the schools founded by the first generations of Jesuit priests and he authored a version of Euclid's *Elements* that was used in Jesuit schools all over Europe. Clavius was also a key figure in the development of our current Gregorian calendar system.

Matteo Ricci, S.J. (a student of Clavius) was one of the early Jesuit missionaries in China. He facilitated a rich scientific, calendrical, mathematical, and religious exchange between Europe and the last of the Ming dynasty emperors of China. He worked with Chinese scholars and produced a Chinese translation of part of Euclid. (He was also well-known for his techniques of committing information to memory and a biography called *The Memory Palace of Matteo Ricci* by Jonathan Spence is a fascinating, though non-chronological, portrait of Ricci and his time in China.) Those contacts continued after Ricci's death through contacts between other Jesuits including Johann Schreck, S.J. and Johann Adam Schall von Bell, S.J. and emperors from the Qing dynasty.

Before entering China, Ricci also worked briefly in the Jesuit mission in Goa in south India, where other Jesuit mathematicians had contacts with Indian mathematicians. It was this connection that George G. Joseph posits as plausible evidence for transmission of Indian precursors of ideas from calculus from Kerala to Europe in the early 1600's in *The Crest of the Peacock* and other subsequent works. David Mumford's somewhat more technical review of the book *Mathematics in India* (authored by Kim Plofker) in

an issue of the *Notices of the American Mathematical Society* may also be of interest, although it does not deal with the suggested "Jesuit connection." *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook* is also an excellent resource for these topics. Because of the level of the mathematics involved in this aspect of the topic (infinite series, Taylor series of functions, and so on), I would recommend a possible project about the suggested precursors of calculus topics in Kerala mathematics only for students who have completed or are currently enrolled in MATH 134, or 136 or the equivalent (Calculus 2).

There is material for several distinct project topics here – three different ones at least. The first step would be to narrow down the focus to something suitable for your project. If you are interested in pursuing a part of this, discuss the options with me before starting work on your bibliography.

Area 2 – Mathematics, the arts, logic and other connections

Another sort of "mathematical thinking" can be seen in the art of M.C. Escher. We looked at the symmetry groups of symmetric strip patterns last semester. Escher used many related ideas in a series of drawings illustrating "tilings" of the plane by repeating motifs. These drawings were just one facet of his work; many other aspects were also mathematically-inspired. One nice project topic would be to investigate some of the mathematical aspects of Escher's work and the extent to which he was aware of and in contact with the work of professional mathematicians. To what extent was he really "doing mathematics" or "thinking mathematically" in his art? Several books and articles of Doris Schattschneider would be the best places to start for this. Her book *Visions of Symmetry* is a great source for Escher's symmetry notebooks. Her article *"The Mathematical Side of M.C. Escher"* from an issue of the *Notices of the American Mathematical Society* is also very interesting.

I first learned about many of the ideas connecting Escher, Bach, mathematics, and so on from another wonderful book called *Gödel, Escher, Bach* by Douglas Hofstadter. Kurt Gödel was one of the great *logicians* of the 20th century. One of his most important contributions was a proof of a result called the Incompleteness Theorem. This work has had a tremendous influence on the way some people (mainly philosophers) think about mathematics. The Incompleteness Theorem says, in essence, that in *every axiomatic mathematical system* that is "powerful" enough to describe the basic properties of addition and multiplication in the integers, there are *true statements that cannot be proved to be true, and false statements that cannot be proved to be false.* (In a sense, you might think of this as a refutation of the whole idea of Euclidean mathematics – any system like Euclid's axiomatic development of number theory in the later books of the *Elements* is necessarily incomplete in the sense that deductive proof cannot uncover all the true statements about numbers.) The idea he used to derive this rather unexpected and unpleasant (at least for mathematicians) result was a clever use of a sort of *self-reference.* In intuitive terms, he showed there were ways to "encode" mathematical statements and proofs as integers, and hence that it is possible to work paradoxical statements like "this statement is false" (think about that one!) right into the "fabric" of the theory of numbers. Hofstadter discusses this in a very lively style, and draws connections between Gödel's proof and some Escher

prints and Bach music. Hofstadter also proposes that perhaps this sort of self-reference might be the key to creating *artificial intelligence* – a hot topic back in the 1980's when this book was written, and an idea whose time may be coming again. It's a very "heady" mixture, but understanding it is definitely worth the effort.

Area 3 – More on Islamic Mathematics

In class we will discuss some of George G. Joseph's ideas about the ways Islamic mathematicians united and developed the legacies of Greek (and Indian) mathematics during Europe's "Dark Ages" and then were instrumental in the reintroduction of those ideas to Europe during the Renaissance period. However, due to limitations of time, we will not be able to touch on a number of important parts of this historical arc. There are several different aspects of this story that would make for very good final project topics. First, the establishment of the library and translation center known as the Bait al-Hikma ("House of Wisdom") in Baghdad during the rule of the Abassid caliphs was instrumental in preserving the classic Greek texts for later scholars. There is a recent book called *The House of Wisdom* by Jonathan Lyons that deals with this. A slightly older book called *Aladdin's Lamp: How Greek Science Came to Europe Through the Islamic World* by John Freely covers a lot of the same ground. Both of these books are quite complete, but somewhat dry, discussions of who did what and when they did it.

In addition to the scholarly center at Baghdad, the Islamic civilization of Al Andalus in present-day Spain became another great center of learning under the Umayyad caliphs and their successors. In particular, many of the translations of Greek classics into Latin (which hastened their reintroduction into Europe) were done in the city of Toledo in a rather amazing collaboration of Islamic and Christian scholars in the 1100's and 1200's. The two books mentioned above in this area deal with some of this story. Another book that deals with the context for this contact and with the story of Al Andalus in general is *The Ornament of the World* by Maria Rosa Menocal. The BBC TV series "When the Moors Ruled in Spain" from 2005 (available in several different versions on YouTube) is another (but inevitably slightly superficial) treatment. This aspect of this group of topics would be great for anyone who wants to learn more about the culture that produced the Alhambra in Granada and its mathematical accomplishments.

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook is also an excellent resource for the actual mathematics connected with these topics.

Area 4 – More on Chinese Mathematics

In some cases, Chinese mathematicians discovered results and techniques that were only developed much later in Europe. Moreover, there were more extensive trade contacts between China and India, and between India and Europe. So the whole development of our modern understanding of mathematics almost certainly involves more exchanges of ideas than than traditional histories often acknowledge. There are a number of slightly more technical mathematical topics from Chinese history that would be interesting for people who might want to explore this area. Two ideas in particular that might form the basis for good projects are

1. the result called the *Chinese Remainder Theorem* concerning simultaneous congruences modulo several different integers, and
2. Chinese techniques for solving simultaneous systems of linear equations that anticipate the modern ideas of matrix algebra and Gaussian elimination.

Either one of these (as well as several other possible mathematical topics) would be acceptable as a project by itself. So there are at least two different possible topics here. *The Crest of the Peacock* has good discussions of both of these, so that is probably the best place to start. *The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook* is also an excellent resource for these topics. In each case, the idea would be to explain the cultural and historical context for the mathematics, then work through the details of what the Chinese actually did, together with its relation to modern methods.

Area 5 – What do Mathematicians Think about Mathematics?

The beginning chapters of the book *Loving and Hating Mathematics* by Hersh and John-Steiner are a rich source if ideas for more open-ended projects about different topics related to the culture of research mathematicians, how different mathematicians think about the subject and why they find it attractive, how they collaborate with colleagues (or not), etc. Another interesting book they mention several times is the memoir *A Mathematician's Apology* by G.H. Hardy, a well-known British mathematician from the late 19th and early 20th century. Hardy has some very provocative ideas about why humans do mathematics and what its ultimate purpose is. This topic (group) is probably most appropriate for the more philosophically-minded. Some thought will be necessary here to narrow the focus down to a manageable project topic. I would be happy to discuss options with you if you would like that.

Area 6 – A Topic of Your Choice

If there is another topic you would prefer to work on, I am open to suggestions. If you want to propose a topic of your own, you *must get my approval* before starting to work. For the March 17 deadline above, write up a short description of the topic or questions you want to look at and how you want to try to address them. I will let you know as soon as possible whether you have my approval.