MONT 107Q – Thinking About Mathematics
Discussion on Diophantus
March 1 (and 3?), 2017

Background

The following are roughly literal translations of the next four Propositions in Book I of the *Arithmetica* of Diophantus after the ones in the reading for today. The goal in this exercise is to "decode" what Diophantus is saying, translate it into modern algebra and think about how you would go about solving similar problems even if the data involved different given numbers. Does what Diophantus writes give enough of an idea of how to solve the problem in general?

Each group will be responsible for presenting their work *orally to the class* later in the period on Wednesday, or possibly in class on Friday if more time is required.

Proposition 7. From the same [unknown] number, subtract two given numbers leaving the remainders in a given proportion.

Let it be proposed to subtract 100 and 20 from some number leaving numbers with the larger in 3^{pl} ratio to the smaller. Let the number sought be laid out as $\varsigma 1$. The remainder on subtracting 100 is $\varsigma 1 \Lambda M^o 100$ and the remainder on subtracting 20 is $\varsigma 1 \Lambda M^o 20$. We must have the greater to the smaller in 3^{pl} ratio, so the larger is three times the smaller, so $\varsigma 3 \Lambda M^o 300$ and $\varsigma 1 \Lambda M^0 20$ are the same. Let the lack be added to both, so $\varsigma 3$ and $\varsigma 1 M^o 280$ are the same. Now let equals be subtracted from equals, and $\varsigma 2$ and $M^o 280$ are the same. Therefore ς is 140.

To what was set down ... If 100 is subtracted from 140, the remainder is 40. If 20 is subtracted from 140, the remainder is 120 which is three times 40. So the larger is three times the smaller.

Proposition 8. Add to two given numbers the same [unknown] number to make the sums have a given proportion.

It is necessary for the given proportion to be smaller than the proportion between the larger and the smaller given numbers.

Let it be proposed to add $M^o 100$ and $M^o 20$ to a number yielding sums in 3^{pl} ratio. Let the unknown number be $\varsigma 1$. Then the sums are $\varsigma 1 M^o 100$ and $\varsigma 1 M^o 20$. Since the larger is to be three times the smaller, $\varsigma 1 M^o 100$ and $\varsigma 3 M^o 60$ are the same. Take away equals from equals. The remainders are $M^o 40$ and $\varsigma 2$ and so ς becomes $M^o 20$.

To what was set down. If 100 is added to 20, the sum is 120. If 20 is added to 20, the sum is 40. And 120 is three times 40. So the larger is three times the smaller.

Proposition 9. From two given numbers subtract the same [unknown] number to make the remainders have a given proportion.

It is necessary for the given proportion to be larger than the proportion between the larger of two given numbers and the smaller one.

Let it be proposed to subtract the same number from 20 and 100 leaving remainders that are in 6^{pl} ratio, the greater to the smaller. Let the number to be subtracted from the two given numbers be $\varsigma 1$. The remainders are then $M^o 20 \Lambda \varsigma 1$ and $M^o 100 \Lambda \varsigma 1$. We must have that the larger is 6 times the smaller, so $M^o 100 \Lambda \varsigma 1$ and $M^o 120 \Lambda \varsigma 6$ are equal. Let the lack be added to both and let equals be subtracted from equals. Then $\varsigma 5$ and $M^o 20$ are equal, so ς is $M^o 4$.

To what was set down. Let the number to be subtracted be 4. Then taking 4 from 20 leaves 16. Taking 4 from 100 leaves 96. And 96 is 6 times 16. So the larger is 6 times the smaller.

Proposition 10. From the larger of two given numbers subtract a[n unknown] number and add the same number to the smaller of the two given numbers to make the sum have a given ratio with the difference.

Let it be proposed to add a number to 20 and subtract the same number from 100 to yield numbers in 4^{pl} ratio, the larger to the smaller. Let the number to be added and subtracted be laid out as $\varsigma 1$. If we add $\varsigma 1$, the sum is $\varsigma 1 M^o 20$; if we subtract the difference is $M^o 100 \Lambda \varsigma 1$. Then $M^o 400 \Lambda \varsigma 4$ and $\varsigma 1 M^o 20$ are equal. Let the lack be added to both and subtract equals from equals. Then $\varsigma 5$ is equal to $M^o 380$ and $\varsigma 1$ is $M^o 76$.

To what was set down. If 76 is added to 20, the sum is 96. If 76 is subtracted from 100, the difference is 24. And 96 is 4 times 24. So the larger is four times the smaller.