MATH 110-02 – Algebra Through History
Discussion 2 – October 7, 2019
Another Proposition from Diophantus

Let's go on and look a bit at a problem in one of the later books of the *Arithmetica*. Here's Proposition 1 in Book II:

Book II, Proposition 1. To find two numbers whose sum has a given ratio to the sum of their squares.

Let it be proposed that the sum of the squares is in 10^{pl} ratio with the sum. (Comment: Diophantus actually states this the opposite way saying the sum is the 10th part of the sum of the squares. That's another way of saying the same thing of course.) Further let it be proposed that the smaller of the two numbers is ς and the larger is $\varsigma 2$. Then the sum is $\varsigma 3$ and the sum of the squares is $\Delta^{\Upsilon} 5$. We must have that $\Delta^{\Upsilon} 5$ and $\varsigma 30$ are the same. So ς is $M^o 6$. The other number is $M^o 12$ and the problem is solved. [Note: As Diophantus progresses through the *Arithmetica*, he writes less and less; presumably his student Dionysius or someone else doesn't need as much detail as they progress in the subject.]

A) Translate Diophantus' argument into modern algebra and show that his method is correct for these numbers. (That is, show that the sum of the squares of the two numbers is 10 times the sum of the two numbers.) In particular, be sure to explain how you are interpreting the relation between ς and Δ^{Υ}. How are those numbers connected?

B) The big question in your mind should be: where did the assumption "the larger is $\varsigma 2$" come from? What would happen if the two numbers were in some other ratio? Say the larger is ςa for some given number a? Can you still solve the problem? Can you always solve it *xwith whole numbers*?

Assignment

Group writeups due in class on Friday, October 11.