MATH 110-02 – Algebra Through History
Thinking About Algebra
September 4, 2019

Background

From the *Los Angeles Times*, 1/30/2006: "In the fall of 2004, 48,000 ninth-graders [in Los Angeles public schools] took beginning algebra; 44% flunked, nearly twice the failure rate as in English. Seventeen percent finished with Ds. In all, the district that semester handed out Ds and Fs to 29,000 beginning algebra students ... Among those who repeated the class in the spring, nearly three-quarters flunked again." These statistics were and are repeated in many school districts across the country. This failure rate has big consequences, of course, because it is very hard for those students to get over the "algebra hurdles" needed for college admission, the later math courses needed for STEM majors and good jobs, etc. You can ask whether algebra is really necessary for those jobs, blame bad teaching, badly run schools, and so forth. But the question we want to consider today is: *Why* is algebra so hard? A first answer, of course, is that it's significantly more *abstract* – not about calculating with numbers as in arithmetic, but about quantifying relationships and solving equations. Some students may find algebra doesn't connect with their everyday experience and it's hard for that reason alone. But let's dig deeper too and try to identify what it is about algebra that's provoking those reactions.

Think-Pair-Share Questions

A. Is it the *symbolic language of algebra*? As we will see, by the time of Viète (mid 1500's C.E.), the use of symbols (for instance, letters) to represent quantities and other symbols to represent relations and operations in algebraic expressions and equations was well advanced and this has continued down to the present! However, we don't always appreciate *just how ambiguous those symbolic expressions can be (especially to learners)* and hence students are not always taught well how to interpret them. Consider each of the following expressions

$$A = L \times W$$
$$5x^2 + 3 = 29x$$
$$\sin(x) = \tan(x)\cos(x)$$
$$1 = x(1/x)$$
$$y = x^3 + 3x + 4$$

1. All five have equals signs. If you translate them into English, is the $=$ sign a noun or a verb or something else? Do the $=$ signs *mean the same thing* in all of them?
2. Four of these have the letter x (the first one does not; confusing the "times" symbol for multiplication with x is an even more basic problem we won't try to address!). Does it mean the same thing in all of them?
3. In the equation $5x^2 + 3 = 29x$ in particular, what *is* x? That's not a direction to solve the problem: I mean, philosophically, *what is it*?

B. Is it the *difficulty of translation* from natural language to symbolic algebra? Solve this problem: *A wallet contains* \$460 *in* \$5, \$10, *and* \$20 *bills. The number of* \$5 *bills exceeds twice the number of* \$10 *bills by* 4, *while the number of* \$20 *bills is* 6 *less than the number of* \$10 *bills. How many bills of each type are there?* Did you go "top down" or "bottom up?"