Mathematics 351 – Abstract Algebra 1
Information on Final Exam
December 3, 2007

General Information, Format and Groundrules

The final exam for this course will be given during the regular scheduled period for 11:00 MWF classes – 8:30am - 11:30am on Wednesday, December 12. This will be an *closed book* exam. The exam will be written so that if you are well prepared and work steadily, you should take about 2 hoursto complete it, but you will have the full 3-hour exam period to work on the exam, so this should eliminate most of the element of time pressure.

Topics

The final exam will be a *comprehensive exam*. It will cover all of the material we have discussed since the beginning of the semester, giving roughly equal attention to the sections of the class corresponding to the two midterms, and also including some of the material from the last sections of Chapter 7 and the beginning of Chapter 8 in Hungerford from the last problem set.

Like the midterm exams, this one will focus mainly on the key definitions and theorems we have studied. There will also be a few problems or parts of problems where you will need to apply some of those definitions and theorems to new situations.

The topics to be covered are:

1) The definition of a ring and key examples.
2) Basic properties of rings.
3) Isomorphisms and homomorphisms.
4) Polynomial rings, the division algorithm and consequences.
5) Divisibility in $F[x]$.
6) Irreducible polynomials and unique factorization.
7) Roots of polynomials and factorizations.
8) Results on irreducibility of polynomials in $\mathbf{Q}[x]$, $\mathbf{R}[x]$, $\mathbf{C}[x]$.
9) Congruence in $F[x]$ and congruence classes.
10) Computing in $F[x]/(p(x))$ via congruence class addition and multiplication.
11) $F[x]/(p(x))$ is a field if and only if $p(x)$ is irreducible in $F[x]$.
12) Ideals, congruence mod I.
13) Quotient rings and homomorphisms, the First Isomorphism Theorem for rings.
14) Prime and maximal ideals, the structure of R/I if I is prime or maximal.
15) Groups, basic properties (such as orders of elements, etc.) and examples, subgroups (cyclic subgroups and others). Be prepared for a question in which you are given the operation table for a group you have not seen before, and you are asked to show things about it.
16) Homomorphisms and isomorphisms of groups.

17) Congruence, cosets, the index of a subgroup, Lagrange's Theorem and its consequences from section 7.5.
18) Normal subgroups, quotient groups, the First Isomorphism Theorem for groups, and applications.
19) The symmetric and alternating groups, disjoint cycle decompositions.
20) Direct products of groups and examples.

Review Session

I would be happy to schedule an evening review session before the exam.

Key Theorems and Proofs to Know

1) Know how to show that a given subset of a ring R is, or is not, a subring of R. Same for showing a given subset of a ring is or is not an ideal.
2) Know how to prove the uniqueness of the quotient and the remainder in polynomial division, given the existence.
3) Know the statement and proof of the Unique Factorization Theorem in $F[x]$ (see Theorem 4.13 in Hungerford, and the class notes).
4) Know how to prove that $a \in F$ is a root of $f \in F[x]$ if and only if $(x - a)|f$.
5) Know the statement and the proof of the Rational Root Test (Theorem 4.20 in Hungerford and the class notes).
6) Know the statement and proof of Eisenstein's Irreducibility Criterion in $\mathbf{Q}[x]$ (see Theorem 4.23 in Hungerford, and the slightly different proof in the class notes).
7) Know how to prove the equivalence of the following statements: a) $p(x)$ is irreducible in $F[x]$; b) $F[x]/(p(x))$ is a field; $F[x]/(p(x))$ is an integral domain. This is Theorem 5.10 in Hungerford; you will find a full proof in the class notes.
8) Know how to prove that a given mapping $f : R \to S$ is or is not a ring homomorphism or that a given mapping $g : G \to H$ is a group homomorphism.
9) Know the statement and proof of the First Isomorphism Theorem for rings (Theorem 6.13 in Hungerford) and for groups (Theorem 7.42 in Hungerford, and the full proof in the class notes).
10) Know how to prove that R/I is an integral domain if and only if I is a prime ideal (Theorem 6.14 in Hungerford).
11) Know how to prove that a given set with a single binary operation is or is not a group. Know how to prove that a given subset of a group is or is not a subgroup. Know how to prove that a give mapping $f : G \to H$ is or is not a group homomorphism.
12) Know the statement and proof of Lagrange's Theorem (Theorem 7.26 in Hungerford).

Philosophical Comments and Suggestions on How to Prepare

- The reason we give final exams in almost all mathematics classes is to give students a final opportunity to "put whole courses together" in their minds. Also, preparing for the final should help to make the ideas "stick" so you will have the material at your fingertips for the second semester of the course (if you are continuing). We will need to use almost everything we have studied this semester at some point next term.

- Get started reviewing early and do some work on this *every day* between now and the date of the final. Don't try to "cram" at the end.
- There are many connections between different portions of what we have done – identifying them and understanding the analogies is a good thing to do at some point.
- Reread your class notes in addition to the text, especially for topics where you lost points on the midterms. There are a lot of worked-out examples and discussions of all of the topics we have covered there.
- Look over the midterm exams with the solutions. Go over your corrected problem sets. If there were questions where you lost a lot of points, be sure you understand why what you did was not correct, and how to solve those questions.
- Be sure you actually do enough practice problems so that you have the facility to solve exam-type questions in a limited amount of time. *Even if you have saved solutions for practice problems from the midterms*, it is going to be much more beneficial to do practice problems starting "from scratch" rather than just reading old solutions.