Mathematics 351 – Abstract Algebra 1
Information on Exam 2
November 7, 2007

General Information, Format and Groundrules

The second midterm exam will be given on Wednesday evening November 14. This will be an *closed book* exam. It will start at 7:00pm, and you will have until 9:00pm to work on it. The exam will be written so that if you are well prepared and work steadily, you should take about 1 hour to complete it, but you will have the extra time to eliminate any time pressure.

Topics

The exam will cover the material we have discussed since the first exam of the semester, up to and including the material on normal subgroups of groups from class on Monday, November 5. This is the material from problem sets 5 through 9: Chapters 5, 6, and 7 up through section 7.6. Of course, much of the material on quotient rings depends heavily on the concepts from Chapters 3 and 4 on basic ring theory, so there will be some overlap with things from the first part of the course.

Like the first exam, this one will focus mainly on the key definitions and theorems we have studied. There will also be a few problems or parts of problems where you will need to apply some of those definitions and theorems to new situations.

The new topics to be covered are:

1) Congruence in $F[x]$ and congruence classes.
2) Computing in $F[x]/(p(x))$ via congruence class addition and multiplication.
3) $F[x]/(p(x))$ is a field if and only if $p(x)$ is irreducible in $F[x]$.
4) Ideals, congruence mod I.
5) Quotient rings and homomorphisms, the First Isomorphism Theorem for rings.
6) Prime and maximal ideals, the structure of R/I if I is prime or maximal.
7) Groups, basic properties (such as orders of elements, etc.) and examples, subgroups (cyclic subgroups and others). Be prepared for a question in which you are given the operation table for a group you have not seen before, and you are asked to show things about it.
8) Homomorphisms and isomorphisms of groups.
9) Congruence, cosets, the index of a subgroup, Lagrange's Theorem and its consequences from section 7.5. (However, we have not yet covered things like Theorem 7.30, so you are not responsible for that.)

Review Session

I would be happy to schedule an evening review session before the exam. Monday, November 12 would probably be the best choice.

Key Theorems and Proofs to Know

1) Know how to prove the equivalence of the following statements: a) $p(x)$ is irreducible in $F[x]$; b) $F[x]/(p(x))$ is a field; $F[x]/(p(x))$ is an integral domain. This is Theorem 5.10 in Hungerford; you will find a full proof in the class notes.
2) Know how to show that a given subset of a ring R is, or is not, an ideal of R. (Note: in most cases, the criterion of Theorem 6.1 in Hungerford is the "preferred method.")
3) Know how to prove that a given mapping $f : R \to S$ is or is not a ring homomorphism.
4) Know the statement and proof of the First Isomorphism Theorem for rings (Theorem 6.13 in Hungerford).
5) Know how to prove that R/I is an integral domain if and only if I is a prime ideal (Theorem 6.14 in Hungerford).
6) Know how to prove that a given set with a single binary operation is or is not a group. Know how to prove that a given subset of a group is or is not a subgroup. Know how to prove that a give mapping $f : G \to H$ is or is not a group homomorphism.
7) Know the statement and proof of Lagrange's Theorem (Theorem 7.26 in Hungerford).

Suggested Practice/Review Problems

From Hungerford:

5.1/6, 9, 10, 11, 12;
5.2/2, 7, 8, 10, 16 (the point of this problem is to prove the field properties *directly*; do not use Theorem 5.10);
5.3/2, 3, 6, 11;
6.1/3, 4, 10, 11, 15, 25 (the kernel!), 27, 29, 41;
6.2/4, 7, 10, 11, 18 (note the connection with Section 6.3!), 21, 23;
6.3/2, 8, 9;
7.1/4, 15, 23, 27;
7.2/11, 12, 13, 14, 23, 34;
7.3/3, 4, 5, 10 ($Z(G) = \{g \in G : gh = hg$ for all $h \in G\}$ is the center of G), 17 ;
7.4/1, 2, 6, 9, 12, 16;
7.5/ 1, 3, 4, 12, 15, 17, 21;