

Mathematics 351 – Abstract Algebra 1  
Information on Exam 1  
September 24, 2007

*General Information, Format and Groundrules*

The first midterm exam will be given Monday evening October 1. This will be an *closed book* exam. It will start at 7:00pm, and you will have until 9:00pm to work on it. The exam will be written so that if you are well prepared and work steadily, you should take about 1 hour to complete it, but you will have the extra time to eliminate any time pressure.

*Topics*

The exam will cover the material we have discussed since the beginning of the semester, up to and including the material on the irreducible polynomials in  $\mathbf{R}[x]$  and  $\mathbf{C}[x]$  from class on Monday, September 24. This is the material from Chapters 3 and 4 in Hungerford.

The exam will focus mainly on the key definitions and theorems we have studied. There will also be a few problems or parts of problems where you will need to apply some of those definitions and theorems to new situations.

The topics to be covered are:

- 1) The definition of a ring and key examples.
- 2) Basic properties of rings.
- 3) Isomorphisms and homomorphisms.
- 4) Polynomial rings, the division algorithm and consequences.
- 5) Divisibility in  $F[x]$ .
- 6) Irreducible polynomials and unique factorization.
- 7) Roots of polynomials and factorizations.
- 8) Results on irreducibility of polynomials in  $\mathbf{Q}[x]$ ,  $\mathbf{R}[x]$ ,  $\mathbf{C}[x]$ .

*Review Session*

We will review for the exam in class on Monday, October 1. Of course this means you will not have much time between then and the exam to do further preparation. *Begin your studying before the review session, please!*

*Key Theorems and Proofs to Know*

- 1) Know how to show that a given subset of a ring  $R$  is, or is not, a subring of  $R$ . (Note: in most cases, the criterion of Theorem 3.6 in Hungerford is the “preferred method.”)
- 2) Know how to show that a given mapping  $\varphi : R \rightarrow S$  is, or is not, a homomorphism of rings. Same for isomorphisms.

- 3) Know how to prove the uniqueness of the quotient and the remainder in polynomial division, given the existence.
- 4) Know the statement and proof of the Unique Factorization Theorem in  $F[x]$  (see Theorem 4.13 in Hungerford, and the class notes).
- 5) Know how to prove that  $a \in F$  is a root of  $f \in F[x]$  if and only if  $(x - a) \mid f$ .
- 6) Know the statement and the proof of the Rational Root Test (Theorem 4.20 in Hungerford and the class notes).
- 7) Know the statement and proof of Eisenstein's Irreducibility Criterion in  $\mathbf{Q}[x]$  (see Theorem 4.23 in Hungerford, and the slightly different proof in the class notes).

*Suggested Review Problems*

From Hungerford:

3.1/5, 19, 38;

3.2/13, 14, 15, 21 (read page 59 first!), 26;

3.3/13, 16, 18, 31;

4.1/5bd, 6, 19;

4.2/7, 8, 10;

4.3/7, 10, 12, 13, 25, 26;

4.4/3, 14, 17;

4.5/1bd, 10, 16, 17, 18.

4.6/2, 3, 6.