Mathematics 352 – Abstract Algebra 2
Information on Exam 2
April 8, 2008

General Information, Format and Groundrules

The second midterm exam this semester will be given on Wednesday evening, April 16. This will be an *closed book* exam. It will start at 7:00pm, and you will have until 9:00pm to work on it. The exam will be written so that if you are well prepared and work steadily, you should take about 1 hour to complete it, but you will have the extra time to eliminate any time pressure.

Topics

The exam will cover the material we have discussed since the first exam this semester, up to and including the material on the Fundamental Theorem of Galois Theory from class on April 9 (Problem Set 9). This is the material from problem sets 6 through 9: Sections 10.5, 10.6, Chapter 15, 11.1, and 11.2. (*Important Note:* You will see, though, that much of this material makes heavy use of concepts from the earlier sections of Chapter 10, and other things we studied earlier in the class, so there is a certain degree of "cumulativeness" built into this material!) Like the other exams, this one will focus mainly on the key definitions and theorems we have studied. There will also be a few problems or parts of problems where you will need to apply some of those definitions and theorems to new situations.

The topics to be covered are:

0) The most important concepts from Sections 10.1-10.4 for this exam are: Extension fields, simple extensions (that is $K = F(u)$ for some single u in K), algebraic and transcendental elements over F, minimal polynomials of algebraic elements, how to find $[F(u) : F]$ and a basis for $F(u)$ over F when u is algebraic over F, algebraic extensions and their properties. Splitting fields of $f(x) \in F[x]$ and normal extensions.
1) Separable polynomials and extensions, the Primitive Element Theorem (Theorem 10.18).
2) Finite fields, the prime field, constructing fields of order p^n for all $n \geq 1$. All fields of order p^n are isomorphic. A field of order p^n has subfields of order p^k if and only if $k \mid n$.
3) Geometric constructions, constructible numbers, the characterization of constructible numbers: α is constructible if and only if $[\mathbf{Q}(\alpha) : \mathbf{Q}] = 2^m$ for some m. The three Greek construction problems and their resolution.
4) F-automorphisms of an extension field K of F, the Galois group $\mathrm{Gal}_F K$
5) The Galois correspondence between subgroups of $\mathrm{Gal}_F K$ and intermediate fields E with $F \subseteq E \subseteq K$, Galois extensions, the Fundamental Theorem of Galois Theory.

Review Session

I would be happy to schedule an evening review session before the exam. Monday, April 14 would probably be the best day for that.

Key Theorems and Proofs to Know

1) Know the statement of the Primitive Element Theorem (Theorem 10.18).
2) Know how to show that the splitting field of $x^{p^n} - x$ over $F = \mathbf{Z}_p$ is a field of order p^n. (This is one implication from Theorem 10.25.)
3) Know the statement of Theorem 15.6 in Hungerford (constructible numbers and quadratic towers), and know how to prove the characterization that if α is constructible, then $[\mathbf{Q}(\alpha) : \mathbf{Q}] = 2^m$ for some integer $m \geq 0$ (see the class notes for this; Hungerford does not state this particular fact). The converse is also true here.
4) Know the proof of Theorem 11.3 in Hungerford: If K is the splitting field of $f(x) \in F[x]$ and u, v are elements of K, then there exists $\sigma \in \mathrm{Gal}_F K$ with $\sigma(u) = v$ if and only if u, v have the same minimal polynomial (know the part of the proof given in the previous Theorem 11.2, and how Theorem 10.14 applies here).
5) Know the statement of the Fundamental Theorem of Galois Theory (Theorem 11.11 in Hungerford). Also know the statement and proof of the Lemma used in the proof of the Fundamental Theorem (Lemma 11.7 in Hungerford).

Suggested Practice/Review Problems

- 10.5/9,11,12b
- 10.6/9,10,16
- Chapter 15/3,5,6,14 (and show that if $c > 0$ is constructible, then $\sqrt{c}$ is also constructible)
- 11.1/1,4,5,6,11,13,14,17
- 11.2/5,6,14
- Exhibit the Galois correspondence for
  (1) $K = \mathbf{Q}(\sqrt[3]{3})$, $F = \mathbf{Q}$
  (2) $K =$ the splitting field of $f(x) = x^3 - 3 \in \mathbf{Q}[x]$, $F = \mathbf{Q}$.