Mathematics 352 – Abstract Algebra 2
Information on Exam 2
February 20, 2007

General Information, Format and Groundrules

The first midterm exam this semester will be given on Wednesday evening, February 27. This will be an *closed book* exam. It will start at 7:00pm, and you will have until 9:00pm to work on it. The exam will be written so that if you are well prepared and work steadily, you should take about 1 hour to complete it, but you will have the extra time to eliminate any time pressure.

Topics

The exam will cover the material we have discussed since the start of the semester, up to and including the material on splitting fields and normal extensions from class on February 20 (Problem Set 5). This is the material from problem sets 1 through 5: Sections 8.2, 8.3, 8.4, 8.5, 10.1, 10.2, 10.3, and 10.4. Like the exams last semester, this one will focus mainly on the key definitions and theorems we have studied. There will also be a few problems or parts of problems where you will need to apply some of those definitions and theorems to new situations.

The new topics to be covered are:

1) The structure theorem for finite abelian groups: Every finite abelian G is isomorphic to a direct sum of cyclic groups of prime power order. Know the classifications of these via the elementary divisors and the invariant factors and how to go back and forth between both forms.
2) The Sylow theorems and the background needed for their proof – the conjugacy relation on a group, the class equation of G and the role of the center $Z(G)$, centralizers of $a \in G$, the formula $|C_a| = [G : C(a)] = |G|/|C(a)|$ (that is: the order of the conjugacy class of a is equal to the index of the centralizer of a), normalizers of subgroups, and so forth.
3) Applications of the Sylow theorems to structure of finite groups, dihedral groups, etc.
4) Vector spaces over a field, including examples where the vector space is an extension field.
5) Extension fields, simple extensions (that is $F(u)$ for u), algebraic and transcendental elements over F, minimal polynomials of algebraic elements, how to find $[F(u) : F]$ when u is algebraic over F.
6) Algebraic extensions and their properties.
7) Splitting fields of $f(x) \in F[x]$ and normal extensions.

Review Session

I would be happy to schedule an evening review session before the exam. Monday, February 25 would probably be the best choice.

Key Theorems and Proofs to Know

1) Know the statement of the structure theorem for finite abelian groups (Theorem 8.7 in Hungerford).
2) Know the statements of the three Sylow theorems and the proof of Sylow 1: If p is a prime and $p^n \| G|$, then G contains a subgroup of order p^n.
3) Know the proof of the multiplicativity of degrees in towers of field extensions: If $F \subseteq E \subseteq K$ are field extensions, and $[E : F]$ and $[K : E]$ are finite, then $[K : F] = [K : E][E : F]$ (Theorem 10.4 in Hungerford).
4) Know the proof of the main theorem on simple extensions (Theorem 10.7 in Hungerford): $F(u) \simeq F[x]/(p(x))$ where $p(x)$ is the minimal polynomial of u over F, and $[F(u) : F] = n$ if $\deg p(x) = n$.
5) Know the proof that $[K : F]$ finite implies every element of K is algebraic over F (Theorem 10.9 in Hungerford).

Suggested Practice/Review Problems

From Hungerford:

8.2/3, 5, 6, 7, 14, 15, 16;
8.3/7, 12, 14, 15;
8.4/5, 8, 14, 15;
8.5/2, 3, 14, 15;
10.1/23, 24, 37;
10.2/8, 9, 10, 11, 21, 22;
10.3/4, 6, 9, 10, 13, 14;
10.4/3, 4, 5, 9, 10, 17;