Mathematics 243, section 1 – Algebraic Structures
Information on Exam 1
September 20, 2012

General Information

As announced in the course syllabus and the schedule on the course homepage, the first exam this semester will be given Friday, September 28. The exam will cover the material we have discussed since the start of the semester, through the material on matrix multiplication from class on September 21. The topics to review are:

1) Sets, set operations: union, intersection, complement, set difference, Cartesian products and their properties
2) Mappings (functions), the domain and range, direct and inverse images, injectivity and surjectivity
3) Binary operations, the properties of associativity, commutativity, existence of identity elements and inverses, what it means for a subset of a set to be closed under a binary operation
4) Composition of functions, identities and inverses for composition
5) Matrix sums and products, and their properties.

Practice Problems

The exam will consist of 4 or 5 problems (each possibly having several parts). Some questions will be computational, some will ask for definitions and/or (short) proofs. The questions will be similar to parts of the following questions from Gilbert and Gilbert:

1) Section 1.1: 5, 19, 23, 31, 33
2) Section 1.2: 3, 4 (be prepared to give complete proofs why functions are 1-1 or onto, or not, in cases like this), 13, 24
3) Section 1.3: 3, 4, 5, 6
4) Section 1.4: 2jm, 3, 10
5) Section 1.5: 1de, 2, 3, 4, 5
6) Section 1.6: 2, 3, 4, 12, 22 (note: the (aa) row can be either the first or the second row of the matrix).

Suggestions on How To Prepare

Read over your notes from class (several times, if necessary!). Make sure you can follow all the steps in proofs and examples. Review the problems from the problem sets, and pay particular attention to any problems you missed the first time.

After you do that, try some of the practice problems above.

When you feel prepared, select four or five of the problems above (maybe just one or two parts for the ones with lots of parts), mixing several computational ones and some proofs. Give yourself a practice exam using those problems, timing yourself to make sure you can work under time pressure.

Review Session

I would be willing to run an evening review session before the exam, but scheduling it will be tricky, since I have other commitments Monday, Tuesday and Wednesday evenings. Thursday would be OK.

Sample Exam

Disclaimer: This was the first exam I gave the last time I taught MATH 243. The exam for this class will be roughly the same length and the same degree of difficulty, but the questions might be different in format and organization.

I. For this problem,

$$\begin{aligned} U &= \mathbf{Z} \\ A &= \{x \in \mathbf{Z} : x \text{ is odd and } -10 \le x \le -4\} \\ B &= \{-2, -1, 0, 1\} \\ C &= \{x \in \mathbf{Z} : x < 0\} \end{aligned}$$

and $f : \mathbf{Z} \to \mathbf{Z}$ is the mapping defined by

$$f(x) = \begin{cases} x + 3 & \text{if } x \text{ is even} \\ x - 1 & \text{if } x \text{ is odd} \end{cases} .$$

A) (10) What is the set $A \cup (B \cap C')$?
B) (10) What is the set $f(B)$?
C) (10) Given: f is a permutation of $\mathbf{Z}$ (you don't need to show this). Find the inverse mapping $f^{-1} : \mathbf{Z} \to \mathbf{Z}$.

II. Let $f : A \to B$ be any mapping.
A) (15) Prove that for any subsets T_1 and T_2 of B, $f^{-1}(T_1 \cup T_2) = f^{-1}(T_1) \cup f^{-1}(T_2)$.
B) (10) Is $f^{-1}(f(S))$ always equal to S for all subsets $S \subseteq A$? If you say no, give a counterexample; if you say yes, say why.

III. Let $*$ be the binary operation on $\mathbf{R}$ defined by $x * y = xy - 2x$.
A) (10) Show with specific counterexamples that $*$ is *not associative*, and *not commutative*.
B) (15) An element e is said to be a *left identity* for a binary operation if $e * x = x$ for all x. Similarly, e is a *right identity* if $x * e = x$ for all x. Does the $*$ operation have a left identity? Does it have a right identity? If so, say what e is in each case. If not, explain how you reach your conclusions.

IV. In both parts of this problem,

$$A = \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix}.$$

A) (10) Compute the matrix product AB or say why it doesn't exist.

B) (10) Define a mapping

$$F : M_{2\times 2}(\mathbf{R}) \to M_{2\times 2}(\mathbf{R})$$

for X a 2×2 matrix by by $F(X) = BX$ (matrix product). Prove that F is one-to-one (injective) and give its left inverse mapping.