Mathematics 243, section 1 – Algebraic Structures
Review Sheet, Final Exam
December 8, 2003

General Information

The final examination for this course will be given at 8:30 a.m. on Tuesday, December 16 in our regular class room, Swords 302. The exam will be roughly twice the length of one of the three midterms, but you will have the full three hour period from 8:30 am to 11:30 am to work on it if you need that much time.

Topics to be Covered

1) Basic logic: Logical connectives and truth tables; implications and their converses, contrapositives, and inverses; quantifiers
2) Sets: set operations (union, intersection, difference, complement, Cartesian product, etc.) and their properties.
3) Mappings: the 1-1 and onto properties, direct and inverse images of sets under mappings,
4) Binary operations: identity elements, inverses, properties such as associativity, commutativity, key examples such as function composition, matrix addition and multiplication, addition and multiplication in $\mathbf{Z}$ and $\mathbf{Z}_n$, etc.
5) Relations: especially equivalence relations and the partition of a set into equivalence classes under an equivalence relation (key example: the congruence mod n relation on $\mathbf{Z}$ – the set of equivalence classes in that case is $\mathbf{Z}_n$).
6) Properties of $\mathbf{Z}$: the Well-Ordering Property, proof by mathematical induction, the division algorithm, divisibility, prime numbers and prime factorizations, the gcd and lcm of two integers, Euclid's algorithm for the gcd.
7) The congruence mod n relation on $\mathbf{Z}$ and the integers modulo n, addition and multiplication in $\mathbf{Z}_n$ and their properties. Applications to cryptography (affine and RSA cryptosystems).
7) Groups: the definition, key examples such as $\mathbf{Z}_n$ under addition, the group of units in $\mathbf{Z}_n$ under multiplication, $GL_2(\mathbf{R})$, the symmetric groups S_n, groups of symmetries of geometric figures, etc.
8) Subgroups of groups: know how to determine whether a given subset of a group is a subgroup. Cyclic subgroups.
9) Cyclic groups, generators, orders of elements, etc.
10) Homomorphisms of groups, kernel and image of a homomorphism, isomorphisms.
11) Cayley's Theorem – know how to find a permutation group isomorphic to a given finite group.
12) Cosets of a subgroup and Lagrange's Theorem for finite groups – know the statement: *If G is a finite group and H is a subgroup, then $o(H)|o(G)$.* Also know what it says about orders of elements in a finite group.

Proofs to Know

See the review sheets for Exams 2 and 3

Suggestions on How to Study

Start by reading the above list of topics carefully. If there are terms there that are unfamiliar or for which you cannot give the precise definition, *learn the definitions now; memorize them if you have to!* You simply cannot answer a question about this material if you do not know what the terms mean. Review the class notes. *Everything on the final will be similar to something we have discussed at some point this semester.* Also look back over your graded problem sets and exams. If there are problems that you did not get the first time around, try them again now. Then go through the suggested problems from the three review sheets. If you have worked these out previously, it is not necessary to do them all again. But try a representative sample "from scratch" – don't just look over your old solutions and nod your head if it looks familiar. You need the practice thinking through the logic of how the solution is derived again!

Review Session

I will be happy to run a review session for the final exam during study week. We can discuss a time in class on Monday, December 8.