Mathematics 136 – Advanced Placement Calculus
Solutions to Ungraded Problems on Problem Set 2
September 22, 2009

2.3/5. We compute:

$$\begin{aligned}\lim_{x\to 8}(1+x^{1/3})(2-6x^2+x^3) &= (\lim_{x\to 8} 1+x^{1/3})\cdot(\lim_{x\to 8} 2-6x^2+x^3) \text{ by Limit Law 4}\\ &= (1+\lim_{x\to 8} x^{1/3})\cdot(2-6\lim_{x\to 8} x^2+\lim_{x\to 8} x^3) \text{ by Limit Laws 1, 3}\\ &= (1+2)(2-6\cdot 64+512) \text{ by Limit Law 6}\\ &= 3\cdot 130 = 390.\end{aligned}$$

2.3/42. We have by Limit Law 11:

$$\lim_{v\to c^-}\sqrt{1-v^2/c^2} = \sqrt{\lim_{v\to c^-}(1-v^2/c^2)} = 0.$$

This says that the length of the object appears (to any observer) to go to zero as its speed relative to that observer approaches the speed of light. There are two ways to answer the last part: Physically, no object can move faster than c, so we must take the limit as $v\to c^-$. Mathematically, the domain of the L function only consists of v with $-c\le v\le c$. So again, the limit must be taken as $v\to c^-$ (or else we get sqare roots of negative numbers).

2.4/6. There are infinitely many correct graphs here. Any graph showing $\lim_{x\to -1^-} g(x) = g(-1)$ and $\lim_{x\to 4^+} g(x) = g(4)$ is OK. For instance we might have jump discontinuities at $x=-1,4$ and the value of g at -1 matches the limit from the left, while the value of g at 4 matches the limit from the right.

2.5/5. Again, infinitely many correct graphs. There should be a vertical asymptote at $x=0$, where $f(x)\to-\infty$ from both sides (notice the form of the limit as $x\to 0$). There should also be horizontal asymptotes at $y=5$ to the left and $y=-5$ to the right.

2.5/26. We have

$$\begin{aligned}\lim_{x\to\infty}\frac{x+2}{\sqrt{9x^2+1}} &= \lim_{x\to\infty}\frac{x+2}{\sqrt{9x^2+1}}\cdot\frac{1/x}{1/x}\\ &= \lim_{x\to\infty}\frac{1+2/x}{\sqrt{9+1/x^2}}\\ &= \frac{1}{3},\end{aligned}$$

since $2/x\to 0$ and $1/x^2\to 0$ as $x\to\infty$.

2.6/17. Of these numbers only $g'(0)$ is negative (the slope of the tangent at 0), $g'(-2)$ looks larger (tangent is steeper) than $g'(2)$ or $g'(4)$. Hence the order is

$$g'(0) < 0 < g'(4) < g'(2) < g'(-2).$$

2.7/9. Since the graph of f is made up of straight line segments, the graph $y = f'(x)$ will be "piecewise constant." Starting from the left, $y = f'(x)$ should be a negative constant c_1 up to $x = -a$ (say), a positive constant c_2 from $x = -a$ to $x = 0$, the negative constant $-c_2$ from $x = 0$ to $x = a$, and $-c_1$ from $x = a$ on. The derivative graph has jump discontinuities at $x = -a, 0, a$, and $f'(x)$ is undefined there (so open circles on both sides on $y = f'(x)$).