Mathematics 136 – Advanced Placement Calculus
Solutions to Ungraded Problems on Problem Set 1
September 15, 2009

1.2/15. (a) We want T as a function of N. The slope of the linear function is given by $\frac{80-70}{173-113} = \frac{10}{60} = \frac{1}{6}$. By the point-slope form, $T - 70 = \frac{1}{6}(N - 113)$, so $T = \frac{1}{6}N + \frac{307}{6}$.

(b) The slope is $\frac{1}{6}$. The meaning of this is that each increase of 6 chirps per minute corresponds to a 1 degree increase in temperature.

(c) If $N = 150$, then $T = \frac{1}{6}(150) + \frac{307}{6} \doteq 76$ degrees Fahrenheit.

1.3/7. The graph $y = \sqrt{3x - x^2}$ is shifted 4 units to the right, reflected over the x-axis and shifted 1 unit down:

$$y = -\sqrt{3(x + 4) - (x + 4)^2} - 1 = -\sqrt{-x^2 - 5x - 4} - 1.$$

1.3/16. The graph $y = \frac{1}{x-4}$ is obtained by shifting the graph $y = \frac{1}{x}$ four units horizontally to the right. (This also shifts the vertical asymptote to the line $x = 4$.)

1.3/24. The graph $y = |\cos(\pi x)|$ is obtained from $y = \cos(x)$ by compressing horizontally (the period becomes 2 rather than 2π), and reflecting the parts of the graph $y = \cos(\pi x)$ that lie below the x-axis over the x-axis. See Figure 1 (on the back page).

1.5/21. Since $(1, 6)$ and $(3, 24)$ must be on the graph $y = Ca^x$, we get the two equations

$$\begin{aligned} 6 &= Ca^1 = Ca \\ 24 &= Ca^3. \end{aligned}$$

Solving for C in the first equation, $C = \frac{6}{a}$. So substituting into the second equation, we get $24 = \frac{6}{a}a^3 = 6a^2$. Since $a > 0$, this shows $a = 2$, and hence $C = 3$. The equation is $y = 3 \cdot 2^x$.

1.6/26. First we solve the equation $y = \frac{e^x}{1+2e^x}$ for x as a function of y:

$$\begin{aligned} (1 + 2e^x)y &= e^x \text{ (after clearing denominators)} \\ y + 2ye^x &= e^x \\ y &= (1 - 2y)e^x \\ \frac{y}{1 - 2y} &= e^x \\ \ln\left(\frac{y}{1 - 2y}\right) &= x \text{ (after taking logarithms to solve for } x\text{)} . \end{aligned}$$

Then $f^{-1}(x) = \ln\left(\frac{y}{1-2y}\right)$ is the inverse function (swap x and y).

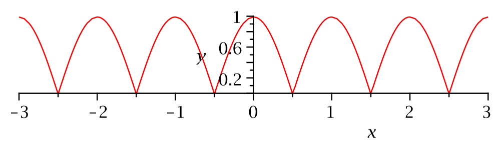

Figure 1: Plot for 1.3/24

1.6/51. Solve for x in the exponent by taking logarithms. Using natural log,

$$\begin{aligned}\ln\left(2^{x-5}\right) &= \ln(3)\\ (x-5)\ln(2) &= \ln(3)\\ \ln(2)x &= \ln(3) + 5\ln(2)\\ x &= \frac{ln(3) + 5\ln(2)}{\ln(2)} = \frac{\ln(96)}{\ln(2)},\ \text{or } 5 + \frac{\ln(3)}{\ln(2)}.\end{aligned}$$

2.2/5 (a) $\lim_{t\to 0^-} g(t) = -1$.
(b) $\lim_{t\to 0^+} g(t) = -2$.
(c) $\lim_{t\to 0} g(t)$ does not exist since the one-sided limits are different.
(d) $\lim_{t\to 2^-} g(t) = 2$.
(e) $\lim_{t\to 2^+} g(t) = 0$.
(f) $\lim_{t\to 2} g(t)$ does not exist (same reason as in (c)).
(g) $g(2) = 1$
(h) $\lim_{t\to 4} g(t) = 3$.

2.2/15. There are infinitely many correct graphs here. They would all have jump discontinuities at $x = 3$, and removable discontinuities at $x = -2$.