Mathematics 136, section 2 – AP Calculus
Information on Final Exam
December 7, 2009

Background

The final exam for this section of AP Calculus will be given *at 8:30 a.m. on Friday, December 18, in our regular classroom – Stein 316*. The exam will be *comprehensive*, covering all the material we have studied since the beginning of the semester. In addition to the material covered on the three midterms, this includes the topics of geometric and Taylor series from the last week.

I will write the exam to be roughly twice the length of the midterms. However, you will have the full three-hour period 8:30 - 11:30 a.m. to work on the exam if you need that much time. The questions will be mostly similar to those on the 3 in-class exams, although there might be some that combine topics in somewhat different ways (see II, IV in the Practice Final below). I may include some questions asking for definitions of terms or statements of theorems (see III in the Practice Final below).

Topics to be Covered

1) Functions defined by graphs, tables, formulas, the "library of functions"
2) Derivatives (know the definition): know how to sketch the graph of the derivative of a function defined by a graph, how to approximate values of the derivative given a table for the function, how to use the derivative rules on functions defined by formulas.
3) The meaning of the signs of $f'(x), f''(x)$.
4) Differentiability = local linearity, tangent lines, etc.
5) Total change of a function, Riemann sums, the definite integral (know the definition), antiderivatives
6) The Fundamental Theorem of Calculus, and
7) Integration by substitution, by parts, by trigonometric substitution, by partial fractions, using the table. A copy of a suitable portion of the table of integrals from the text will be provided with the exam; any integral you need to compute will be do-able by some combination of substitution, integration by parts, trigonometric substitution, partial fractions, and/or consultation with the table).
8) Applications of integration (setting up problems via Riemann sums; in limit a definite integral is obtained) – volumes by slices, arclengths, physical examples like mass from non-constant density functions, center of mass of a wire or a plate, work.
9) Differential equations – slope fields and solutions
10) Solving differential equations via separation of variables and integration, including population growth models.
11) Geometric series, Taylor series, Taylor polynomials, approximations

Philosophical Comments and Suggestions on How to Prepare

- The reason we give final exams in almost all mathematics classes is to encourage students to "put whole courses together" in their minds. Also, preparing for the final should help to make the ideas "stick" so you will have the material at your disposal to use in later courses.

- It may not be necessary to say this, but here goes anyway: *You should take this exam seriously* – it is worth 30% of your course average and it can pull your course grade up *or down* depending on how you do.
- Get started reviewing early and do some work on this *every day* between now and the date of the final. Don't try to "cram" at the end. There's too much stuff that you need to know to approach preparing that way!
- Reread your class notes in addition to the text, especially for topics where you lost points on the midterms. There are a lot of worked-out examples and discussions of all of the topics we have covered there.
- Look over the midterm exams with the solutions. Go over your corrected problem sets. If there were questions where you lost a lot of points, be sure you understand why what you did was not correct, and how to solve those questions.
- Be sure you actually do enough practice problems so that you have the facility to solve exam-type questions in a limited amount of time.
- Look over some of the problems from the text from the previous review sheets, and the following problems for item 11 above: p. 572-3/13-18, p.616/5,7,9,11,13,15. Also be sure you understand how you can use a Taylor polynomial to approximate a value of a function at an x close to the a used to construct the polynomial.
- Then take a couple of hours and try the practice exam problems below.

Practice Exam

I. Given a graph $y = f(x)$ for some function:
 A) Sketch the graph $y = 2f(x-2)+1$.
 B) Sketch the graph $y = -f(-x)$

(*Note: Be prepared for any problem of this type – i.e. for any graph I might give you.*)

II. Given a graph $y = f(x)$ (*again, be prepared for any graph I might give you*):
 A) Sketch the graph $y = f'(x)$ qualitatively. Show any points where $f'(x)$ does not exist by hollow circle(s) at those x's.
 B) Let $g(x)$ be the *antiderivative* of $f(x)$ with $g(0) = 0$. Sketch the graph $y = g(x)$ qualitatively.

III.
 A) What is the precise definition of the *derivative* of a function f at $x = a$?
 B) What is the precise definition of the *definite integral* of a function f over an interval $[a, b]$?
 C) Give a precise statement of the Fundamental Theorem of Calculus (both parts).

IV. A rectangular metal plate of uniform thickness extends from $x = 0$ to $x = 6$ and $y = 0$ to $y = 4$. The density of the metal decreases with x according to the following table:

x in inches	0	1	2	3	4	5	6
density in grams/square inch	200	190	170	140	100	90	80

(The density is independent of y.)

A) Approximate the total mass of the plate using a suitable Riemann sum.

B) Do you think your estimate is larger or smaller than the true mass? Explain, including any assumptions you are making.

V. Compute the following integrals.

A) $\int_0^1 \cos(\pi x) - x^2 e^{x^3}\, dx$

B) $\int x^2 \sin(3x)\, dx$ (Use parts; check work with the table entry for the appropriate reduction formula.)

C) $\int \frac{x}{(x+3)(x^2+1)}\, dx$ (Use partial fractions.)

D) $\int \frac{dx}{\sqrt{x^2-16x+65}}$ (Complete the square under the radical, then do an appropriate trigonometric substitution.)

VI. Both parts of this question refer to the region R in the first quadrant bounded by the graph $y = 4 - x^2$, the x-axis, and the y-axis.

A) Find the volume of the solid obtained by revolving R around the x-axis.

B) Same, but revolving around the line $x = 3$.

C) Find the coordinates of the centroid of R.

VII. All parts of this question refer to the differential equation

$$\frac{dy}{dx} = xy.$$

A) Sketch the slope field for this equation, showing the slopes at points with $xy = -2, -1, 0, 1, 2$.

B) Solve the differential equation to obtain a formula for $y(x)$ satisfying $\frac{dy}{dx} = xy$ and $y(0) = 4$.

VIII. Newton's Law of Cooling (and Heating) states that if an object is placed in a surrounding medium of constant temperature A, then the rate of change of the object's temperature T with respect to time t is proportional to the difference $T - A$.

A) State Newton's Law of Cooling as a differential equation.

B) Your differential equation in part A should be separable. Solve it to get an explicit formula for T as a function of t.

C) When a batch of Christmas cookies is taken out of the oven it has temperature $T(0) =$ 200 degrees Fahrenheit. The sheet is placed in a room heated to a constant 70 degrees Fahrenheit. 5 minutes later the cookies have cooled to 170 degrees F. When will the cookies reach 120 degrees F and be cool enough to eat with burning your mouth?

IX. A large punchbowl is a hemisphere with radius .5 meters. If the punchbowl is full of Christmas punch with density 950 kg per cubic meter, how much work will be done in emptying the punchbowl by "pumping" the contents out the top? (The acceleration of gravity is $g = 9.8$ meters per second squared.)

X.

A) Does the geometric series

$$1+\frac{e}{\pi}+\left(\frac{e}{\pi}\right)^2+\cdots$$

converge? If so, what is its sum?

B) For which p does the improper integral $\int \frac{1}{x^p}\,dx$ converge?

C) Find the Taylor series of $g(x)=e^{2x}$ at $a=0$.

D) Compute the Taylor polynomial $p_5(x)$ of $f(x)=\sin(x)$ at $a=0$, and use it to approximate $f(.6)$. How close is the approximation to the value of $\sin(.6)$ given by your calculator? (Note: Your calculator must be set to compute the sine of an angle *in radians* for this problem to make any sense!)

Review Session

I will be happy to run an evening review session for the final during exam week if there is interest. We can discuss this in class on Monday, December 7.