Mathematics 136 – Advanced Placement Calculus
Exam 3 – Review Sheet
November 23, 2009

General Information

As announced in the course syllabus, the third midterm exam of the semester will be given at 7:00pm on Wednesday, December 2 (the Wednesday after Thanksgiving break). The format will be similar to that of the other midterms.

- The exam will be designed to take an hour but you will have an extra 30 minutes to work and check your solutions.
- You will be given a TI-30 scientific calculator for the exam which does NOT have graphing capabilities so be prepared to answer questions without your personal calculator. (Note: Some of you may have one of these calculators purchased for use in Chemistry courses here. That is also OK.)
- Use of cell phones, I-pods, and all other electronic devices *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack (make sure cell phones are turned off).

What will be covered

The exam will cover the material since the last exam (Problem Sets 7 and 8), *plus* the material on differential equations. In other words, this is the following material from sections 5.10, 6.1, 6.2, 6.3, 6.4, 6.6, and Chapter 7 of Stewart:

1. Improper integrals (know how to tell what makes an integral improper, how to set up the limit to determine if the integral converges correctly, and determine the value if the integral does converge).
2. Areas between curves
3. Volumes of solids with known cross-sections and solids of revolution
4. Arc length of curves
5. Average value of $f(x)$ on an interval $[a, b]$.
6. Work done stretching/compressing a spring, emptying a tank, etc.
7. Computing centers of mass (centroids) for wires, thin plates.
8. Differential equations: solutions, slope fields (be prepared to sketch a simple one by hand and then add qualitative graphs of solutions, separation of variables for analytic solutions, growth and decay problems.

Important Notes:

1. Many of the problems on this exam will require you to set up and compute an integral to find the quantity that is asked for. In addition to knowing how to set up the required integral, *any* of the methods of integration tested on the second exam (i.e. basic rules, u-substitution, integration by parts, trigonometric substitution, partial fractions, or consultation of a table of integrals) might be required to evaluate the integral. An "extract" of the table of integrals will be provided for you to use on the exam, but you may need to do a preliminary substitution or transform your given integral in some way. In other words, this exam is *really a cumulative exam on the second two-thirds of the semester*. Especially if you did poorly on the second exam, you will need to begin your review for this exam by going back and looking at the material from sections 5.1 through 5.8 in Stewart(!)

2. Because of the timing of the Thanksgiving break, I have decided not to give a mandatory Problem Set 9 before the exam. However, you may submit the following problems (also listed below in the Review Problems for the exam)

    - Section 7.3/1 - 17 (odd numbers).
    - Chapter 7 Review/1, 5-8 (these are all separable!), 11, 13, 15, 17.

    as an *optional, Extra Credit problem set*, due Tuesday, December 1. *All necessary work must be shown on these for credit. Note: I will not have enough time to grade these before the exam, but you will get them back graded before the final. Your score on this problem set will be added on top of your regular problem set average, so this can only help, even if you do not do all of them!*

There will be a review for the exam in class on Tuesday, December 1. *Please do not postpone preparing for this exam until after you return from Thanksgiving break – you may not have enough time to get ready!*

Review Problems

Section 5.10/1, 13, 15, 19 (use parts), 21, 23, 25, 29, 31, 55

Section 6.1/1, 3, 5, 7, 9, 11, 29 (place the origin at the center of the large circle, and note that the endpoints of the arc on the smaller circle are at opposite ends of a diameter of that circle), 37

Section 6.2/1, 3, 5, 9, 21 (and evaluate), 29, 39 a (and evaluate)

Section 6.3/5, 7, 9 (look for an algebraic simplification to integrate), 17, 19 (use the table for these)

Section 6.4/1, 3, 5, 13

Section 6.6/Any question I ask here will be similar to the questions from Problem Set 8.

Section 7.3/1 - 17 (odd numbers).

Chapter 7 Review/1, 5-8 (these are all separable!), 11, 13, 15, 17.

Sample Exam Questions

This list is *much longer* than the actual exam will be (to give you some idea of the range of different questions that might be asked). Unless otherwise directed, you may use any entry of the Table of Integrals from the text that applies.

I. For each of the following integrals, say why the integral is improper, determine if the integral converges, and if so, find its value.

A) $\int_1^\infty \frac{1}{\sqrt[5]{x}}\, dx$

B) $\int_0^2 \frac{dx}{x^2-7x+6}$

C) $\int_0^\infty xe^{-3x}\, dx$

D) For which values of a is $\int_0^\infty e^{ax}\sin(x)\, dx$ convergent? Evaluate the integral for those a.

II.

(A) Let R be the region in the plane bounded by $y = 3 - x^2$ and the x-axis.

- (1) Sketch the region R.
- (2) Find the area of R.
- (3) Find the volume of the solid generated by rotating R about the x-axis.
- (4) Find the coordinates $\overline{x}, \overline{y}$ of its centroid.

(B) Let R be the region in the plane bounded by $y = \cos(\pi x)$, $x = 0$, $x = 1/2$, and $y = 0$.

- (1) Sketch the region R.
- (2) Find the area of R.
- (3) Find the volume of the solid generated by rotating R about the x-axis.
- (4) Find the coordinates $\overline{x}, \overline{y}$ of its centroid.

III. The height of a monument is 20m. The horizontal cross-section of the monument at x meters from the top is an isosceles right triangle with legs $x/4$ meters. Find the volume of the monument.

IV.

(A) Set up and evaluate the integral to compute the arclength of the curve $y = 3x^2$, $0 \le t \le 2$.

(B) Set up and evaluate the integral to compute the arclength of the curve $y = \frac{1}{6}(x^2+4)^{3/2}$, $0 \le x \le 3$. (Hint: the arclength integral simplifies to a manageable form if you are careful with the algebra.)

V.

(A) Find the average value of $f(x) = \sqrt{1-x^2}$ on the interval $[0, 1/2]$. (Use trigonometric substitution, not the table.)

(B) Find the average value of $f(x) = x\sqrt{1+x^4}$ on the interval $[0, 2]$.

VI. A cylindrical tank 10 meters tall and with base radius 3 meters is full of water (density 1000kg per cubic meter). Find the work done in pumping all the water out the top of the tank.

VII.

(A) Show that for any constant c, $y = x^2 + \frac{c}{x^2}$ is a solution of the differential equation

$$y' = 4x - \frac{2}{x}y.$$

(B) All parts of this question refer to the differential equation

$$y' = y(4-y)$$

(1) Sketch the slope field of this equation, showing the slopes at points on the lines $y = 0, 1, 2, 3, 4, 5$

(2) On your slope field, sketch the graph of the solution of the equation with $y(0) = 1$.

(3) This is a separable equation; find the general solution and determine the constant of integration from the initial condition $y(0) = 1$.

(C) Find the general solutions of the following differential equations

(1) $y' = \dfrac{y}{x(x+1)}$

(2) $y' = \dfrac{\sqrt{1-x^2}}{e^{2y}}$.

(D) Newton's Law of Cooling states that the rate at which the temperature of an object changes is proportional to the difference between the object's temperature and the surrounding temperature. A hot cup of tea with temperature 100°C is placed on a counter in a room maintained at constant temperature 20°C. Ten minutes later the tea has cooled to 76°C. How long will it take to cool off to 45°C? (Express Newton's Law as a differential equation, solve it for the temperature function, then use that to answer the question.)