Mathematics 136, section 2 – Advanced Placement Calculus
Review Sheet for Exam 1
September 23, 2009

*General Information*

The first exam for the course will be given next Thursday, October 1 at 7:30pm *in our regular classroom*. You will have until 9:00pm to work on the exam if you need that much time. (Note: This is one day later than announced in the course syllabus because of the Hanify-Howland Lecture). It will cover the material discussed in class from the start of the semester *through and including* the material on implicit differentiation, derivatives of inverse functions and related rates problems from class on September 23. There will be 4 or 5 problems, some possibly with several parts. Some may ask for a graph or the result of a calculation; others may ask for a precise definition of a term or concept we have used, or a short description or explanation of some phenomenon.

*Calculators*

As announced in the course syllabus, I will be providing a basic scientific numerical calculator (TI model TI-30Xa) for you to use on this exam. These calculators can compute $+, -, \times, \div$ and values of all functions we have talked about, but have no graphing or symbolic computation features.

*Review Session*

Since we are taking an extra day for the exam, class on Tuesday, September 29 or Wednesday, September 30 will be an *optional* review session (choice of which day will be up to the class). We will work some example problems, review the background for some topics, and so forth. I think you will find this to be valuable, but you are not obligated to attend if you feel well-prepared.

*Suggested Practice Problems*

From the text:

- Chapter 1 Review problems: 5, 7, 9-15, 24-26
- Chapter 2 Review problems: 1, limit problems like 3-18, 30, 32, 35, 37ab, 39, 41, 43
- Be able to differentiate any functions like p. 248-9/1 - 36 from the Chapter 3 Review problems (you don't need to do all of these; just make sure you can handle all the different functions involved, can spot which derivative rule(s) apply, etc. Also you might need to compute a derivative like one of these as part of a larger problem.)
- Chapter 3 Review problems: 41, 47ab, 53-60, 65
- Section 4.1/3, 5, 11.

*Sample Exam* (The numbers next to each part are the possible points on that part; they add up to 100. Disclaimer: The actual exam questions may be posed differently, may combine these topics in somewhat different ways, etc.)

I. A. (5) Plot the graph of the piecewise-defined function:

$$\begin{cases} t+1 & \text{if } t < 0 \\ -3 & \text{if } t \geq 0. \end{cases}$$

B. (10) On one set of axes, plot the graphs $y = 2f(t-2)$ and $y = f(-t)+1$. Label which is which.

C. (5) Is $f$ continuous at $t = 0$? Explain why or why not.

D. (5) Is $f$ continuous at $t = 2$? Explain why or why not.

II. A population of wildebeest grows in a habitat that can support no more than $P_1$ animals. At time $t = 0$, the number of wildebeest present is $P_0 < P_1$. The population grows more and more rapidly at first, but then the rate of growth decreases and the population approaches $P_1$, without exceeding it.

A. (10) Sketch a graph of the population of wildebeest as a function of time that fits this description. For full credit your graph should show proper slope and concavity.

B. (10) Give a qualitative graph of the derivative function of your population function.

III. A. (5) What is the exact mathematical definition of the *derivative* of a function $f(x)$ at $x = a$?

B. (10) Use the definition (not the "shortcut rules") to compute the derivative of $f(x) = 4x^2 - 2x + 2$ at a general $x$.

C. (5) What is the equation of the tangent line to $y = 4x^2 - 2x + 2$ at the point $(1, 4)$?

IV. Compute derivatives using the "shortcut rules":

A. (5) $f(x) = 2x^3 + 6x^{2/3} + e^{\pi}$. For which $x$ is $f$ differentiable?

B. (5) $f(x) = \frac{\sin(x)}{1+\cos(x)}$

C. (5) $g(x) = \tan^{-1}(e^x - \ln(x))$

D. (5) Find $\frac{dy}{dx}$ by implicit differentiation if $xy^6 + e^y = 1$.

V. (15) A cube of ice is taken out of a freezer and left on a kitchen counter to melt, losing volume at a rate of 2 cubic inches per minute. At what rate is the side of the cube changing when the volume is 64 cubic inches?