Background

Over the last few days we have been thinking about the definition of the derivative of \( f(x) \) at \( x \):

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.
\]

Of course, if we had to use the definition every time we wanted to compute a derivative, it would be very awkward and time consuming. From your high school calculus course, you should recall a number of methods for computing derivatives of functions defined by formulas. We want to review those today and work some examples. The goal is to finish work on this and hand it in by the end of today’s class meeting.

A. The general rules. Say you know that the derivatives of \( f(x) \) and \( g(x) \) exist at \( x \).

1. Say in words, and with a formula how you would find the derivative of the function \( cf(x) + g(x) \) (\( c \) constant) at \( x \).
2. Say in words, and with a formula how you would find the derivative of the product function \( f(x)g(x) \) at \( x \).
3. Say in words, and with a formula how you would find the derivative of the quotient function \( \frac{f(x)}{g(x)} \) at \( x \).
4. Say in words, and with a formula how you would find the derivative of the composite function \( f(g(x)) \) at \( x \).

B. Use the rules from part A, and derivatives for known functions like powers \( x^n \), exponentials \( e^x \), \( \ln(x) \), \( \sin(x) \), \( \cos(x) \), and so on to find derivatives for each of the following. In each case, show all work, write the calculations carefully as equations of the form \( f'(x) = \cdots \), and say which of the rules from part A you are using.

1. \( f(x) = 6x^3 + 7x^{\frac{1}{2}} - 3x^{-\frac{2}{3}} + \pi^e \).
2. \( g(x) = e^x - \sqrt{x} + \frac{1}{x^7} \).
3. \( h(x) = e^x \cos(x) \).
4. \( k(x) = \frac{x^2 + 1}{x^4 + 2} \).
5. \( \ell(x) = x \ln(5x^2 + 1) \).
6. \( m(x) = \sqrt{e^x + 1} \).
7. \( n(x) = \cos(\sin(x^3)) \).
8. \( p(x) = \left( x + \frac{1}{x} \right)^4 \).