Mathematics 136, section 4 – AP Calculus
Information on Final Exam
December 9, 2003

Background

The final exam for this section of AP Calculus will be given *at 8:30 a.m. on Friday, December 19, in our regular classroom – Swords 302*. The exam will be comprehensive, covering all the material we have studied since the beginning of the semester. In addition to the material covered on the three midterms, this includes the topics of Taylor polynomials and Taylor series from the last week.

I will write the exam to be roughly twice the length of the in-class midterms. However, you will have the full three-hour period 8:30 - 11:30 a.m. to work on the exam if you need that much time. The questions will be similar to those on the 3 in-class exams. I may include some questions asking for definitions of terms or statements of theorems.

Topics to be Covered

1) Functions defined by graphs, tables, formulas, the "library of functions"
2) Derivatives (know the definition): know how to sketch the graph of the derivative of a function defined by a graph, how to approximate values of the derivative given a table for the function, how to use the derivative rules on functions defined by formulas.
3) The meaning of the signs of $f'(x), f''(x)$.
4) Differentiability = local linearity, tangent lines, etc.
5) Total change of a function, Riemann sums, the definite integral (know the definition), antiderivatives
6) The Fundamental Theorem of Calculus, and
7) Integration by substitution, by parts, using the table. A copy of the table of integrals from the text will be provided with the exam; any integral you need to compute will be do-able by some combination of substitution, integration by parts, and/or consultation with the table).
8) Applications of integration (setting up problems via Riemann sums; in limit a definite integral is obtained) – volumes by slices, arclengths, physical examples like mass from non-constant density functions, center of mass of a wire.
9) Differential equations – slope fields and solutions
10) Solving differential equations via separation of variables and integration, including population growth models.
11) Geometric series, power series and the Ratio Test for the interval of convergence
12) Taylor series, Taylor polynomials, approximations

Suggestion on How to Study

Begin by reviewing the class notes. Look at the in-class midterm problems and try to work out solutions for those again, without refering to your previous work. Look over some of the problems from the text from the previous review sheets, and the following problems for items 11 and 12 above: p. 410/1-14, p. 428/11-21; p. 440/1,3,5,7,11,13; p. 451/1,3,5,6,9. Also be sure you understand how you can use a Taylor polynomial to approximate a value of a function at an x close to the a used to construct the polynomial. Then take a couple of hours and try the practice exam problems below.

Practice Exam

I.

A) One of the following tables gives a function that is approximately *exponential*: given by ca^x. Identify which one it is and give values for c and a:

x	0	.2	.4	.6	.8
$g(x)$	3.10	3.40	3.70	4.00	4.30

x	0	.2	.4	.6	.8
$h(x)$	3.10	3.56	4.09	4.70	5.40

B) The following graph shows $y = f(x)$ for some function. Sketch the graph $y = 2f(x-2)+1$. (*Be prepared for any problem of this type*)

II. The following graph shows $y = f(x)$. (*Be prepared for any question of this type*)

A) Sketch the graph $y = f'(x)$ qualitatively. Show any points where $f'(x)$ does not exist by hollow circle(s) at those x's.

B) Let $g(x)$ be the *antiderivative* of $f(x)$ with $g(0) = 0$. Sketch the graph $y = g(x)$ qualitatively.

III.

A) What is the precise definition of the *derivative* of a function f at $x = a$?

B) What is the precise definition of the *definite integral* of a function f over an interval $[a, b]$?

C) Give a precise statement of the Fundamental Theorem of Calculus.

IV. A rectangular metal plate of uniform thickness extends from $x = 0$ to $x = 6$ and $y = 0$ to $y = 4$. The density of the metal decreases with x according to the following table:

x in inches	0	1	2	3	4	5	6
density in grams/square inch	200	190	170	140	100	90	80

(The density is independent of y.)

A) Approximate the total mass of the plate using a suitable Riemann sum.

B) Do you think your estimate is larger or smaller than the true mass? Explain, including any assumptions you are making.

V. Compute the following integrals.

A) $\int_0^1 \cos(\pi x) - x^2 e^{x^3}\, dx$

B) $\int x^3 \sin(3x)\, dx$

C) $\int \frac{x-2}{x^2+6x+10}\, dx$

D) $\int \frac{dx}{\sqrt{x^2-16x+63}}$

VI. Both parts of this question refer to the region R in the first quadrant bounded by the graph $y = 4 - x^2$, the x-axis, and the y-axis.

A) Find the volume of the solid obtained by revolving R around the x-axis.

B) Same, but revolving around the line $x = 3$.

VII. In a chemical reaction, the amount $y = y(t)$ of a certain compound that is present changes over time and satisfies the differential equation

$$\frac{dy}{dt} = k(y-2)(y-6)$$

for some $k > 0$.

A) Taking $k = 1$, sketch the slope field for this equation, showing the slopes at points with $y = 1, 2, 3, 4, 5, 6, 7$.

B) Suppose $y(0) = 5$. What happens to the amount of the compound as $t \to \infty$?

C) Solve the differential equation to obtain a formula for $y(t)$. (Hint: you will need to use an entry from the table of integrals for this. State the one you use for partial credit.)

VIII.

A) Compute the Taylor polynomial $p_5(x)$ of $f(x) = \sin(x)$ at $a = 0$, and use it to approximate $f(.6)$.

B) Find the Taylor series of $g(x) = e^{2x^3}$ at $a = 0$ using any applicable method.

C) For which x in the real numbers does the power series

$$\sum_{k=1}^{\infty} \frac{5^k}{2^k k!} x^k$$

converge?

Review Session

I will be happy to run an evening review session for the final during exam week if there is interest. We can discuss this in class on Tuesday, December 9.