Mathematics 136, section 1 – AP Calculus
Information for Exam 3
November 25, 2003

General Information

As per the syllabus given out at the beginning of the semester, it's almost time for the third midterm exam. This exam will be given the Friday of the week after Thanksgiving, December 5. The format will be similar to that of the other exams this semester.

Topics to be Covered

This exam will cover the material we have discussed since Exam 2, up to the material on geometric and power series and convergence covered in class on Monday and Tuesday, November 24, 25. This includes the applications of integrals and the material on differential equations and population growth models. The topics are:

1) Applications of integration (setting up problems via Riemann sums; in limit a definite integral is obtained) – volumes by slices (Cavalieri's Principle), arclengths, physical examples like mass from non-constant density functions, etc. (A copy of the table of integrals from the text will be provided with the exam; any integral you need to compute will be do-able by some combination of substitution, integration by parts, and/or consultation with the table).
2) Differential equations – slope fields and solutions
3) Euler's Method for numerical solutions of differential equations
4) Solving differential equations via separation of variables and integration,
5) Exponential and logistic population growth models.
6) Geometric series, power series (Sections 9.1, 9.4)

Review Session

If there is sufficient interest, I would be happy to hold a review session in the afternoon or evening of Tuesday, December 2. I have commitments Wednesday and Thursday evenings (orchestra rehearsals in Boston that I cannot miss), so those times are not possible for me. We can discuss this in class on Monday, December 1 and set it up.

Some good review problems

From the text:

- From the review problems for Chapter 8 (p. 397): 7,8 (also be able to set up the integrals for the total mass and center of mass of a wire in a shape like one of these if the density function is given), 9-13,25,31
- From the review problems for Chapter 11 (p. 552): 8-23, 31, 34,35, 36, 42. "Check Your Understanding" problems 8,9,10.

- From Chapter 9/Section 1: 1 - 14, Chapter 9/Section 4: 11-14, From the review problems for Chapter 9: 16, 17.

Sample Exam

Note: This is slighly longer than the actual exam will be. Nevertheless, it may help, as a part of your preparation, to set aside an hour or so and take this as a practice test. Solutions will be distributed via the course homepage so that you can check your work.

I. A wire in the shape of the graph $y = x^2$, $x \in [-1, 2]$ has density $4 - x$ at the point (x, x^2).

A) Set up a Riemann sum approximating the *total mass* of the wire, and explain how you got it.

B) What definite integral computes the total mass?

C) Evaluate your integral (using the table from the text as needed).

II. Both parts of this problem refer to the region R bounded by $y = 1 + x^2$, $x = 0$, $x = 1$, and the x-axis.

A) Find the volume of the solid obtained by rotating R about the x-axis.

B) Find the volume of the solid obtained by rotating R about the line $y = -3$.

III. The population of fish in a lake is attacked by a microscopic water-borne parasite at $t = 0$, and as a result the population declines at a rate proportional to the *square root* of the population from that time on.

A) Express this statement about the rate of growth of the population P as a differential equation.

B) There should be a constant of proportionality, say $-k$, in your equation. Setting $-k = -1$, sketch the slope field for the corresponding differential equation for $t \in [0, 4]$, $P \in [0, 4]$, indicating the slope field segment at each point with integer coordinates.

C) Use $n = 4$ steps of Euler's Method to approximate the value of the solution of your differential equation with $P(0) = 200$ at $t = 1$.

D) Use separation of variables to find the general solution of your differential equation.

E) At $t = 0$ there were 900 fish in the lake; 441 were left after 6 weeks. When did the fish population disappear entirely?

IV.

A) Find the sum of the infinite geometric series

$$3 + \frac{3}{7} + \frac{3}{49} + \frac{3}{343} + \cdots$$

B) Use the Ratio Test to determine the interval of convergence of the power series

$$\sum_{k=0}^{\infty} \frac{2^n x^n}{n!}$$