MATH 136 – Calculus 2

Practice Day on Consequences of FTC, "Part I"

January 31, 2020

Background

Recall that the "FTC, part I" says: If f is continuous on $[a, b]$ and F is any function satisfying $\frac{d}{dx}F(x) = F'(x) = f(x)$ for all x in $[a, b]$ (called an *antiderivative* of f), then

$$\int_a^b f(x)\ dx = F(b) - F(a).$$

We will use the *shortcut notation*: $F(x)|_a^b$ for this difference of values at the limits of integration:

$$F(x)|_a^b = F(b) - F(a).$$

Note that you always put the value $F(b)$ at the *upper limit* first and then subtract the value at the lower limit, $F(a)$. This means that we have a "shortcut method" for computing $\int_a^b f(x)\ dx$ as long as we have or can find a suitable *antiderivative* F – a function with $F'(x) = f(x)$ for the function $f(x)$ we are integrating.

Questions

(1) Verify that

$$\frac{d}{dx}(e^{x^2}) = 2xe^{x^2}.$$

Answer: This follows from differentiation using the chain rule.

(2) Use part (1) to evaluate:

$$\int_0^1 2xe^{x^2}\ dx$$

Answer: Using (1),

$$\int_0^1 2xe^{x^2}\ dx = e^{x^2}\Big|_0^1 = e - 1 \doteq 1.71828.$$

(3) Verify that

$$\frac{d}{dx}(\ln(x^2+5x+6)) = \frac{2x+5}{x^2+5x+6}.$$

1. *Answer:* Differentiate using the chain rule and the derivative formula for $\ln(x)$.

(4) Use part (4) to evaluate:

$$\int_1^2 \frac{2x+5}{x^2+5x+6}\ dx.$$

Answer: Using (3),

$$\int_1^2 \frac{2x+5}{x^2+5x+6}\ dx = \ln(x^2+5x+6)\Big|_1^2 = \ln(20) - \ln(12) = \ln(5/3) \doteq .5108$$

(5) Verify that

$$\frac{d}{dx}\left(\frac{1}{2}(1+\sin(x)\cos(x))\right) = \cos^2(x)$$

(You'll need a trig identity here – ask if you don't recall identities using $\sin^2(x)$ and $\cos^2(x)$!)

Answer: Here's one way (there are others too): By the product rule and the identity $\sin^2(x)+\cos^2(x)=1$, we have:

$$\begin{aligned}\frac{d}{dx}\left(\frac{1}{2}(x+\sin(x)\cos(x))\right) &= \frac{1}{2}(1-\sin^2(x)+\cos^2(x)) \\ &= \frac{1}{2}(2\cos^2(x)) \\ &= \cos^2(x)\end{aligned}$$

(6) Use part (5) to evaluate:

$$\int_0^{\pi/2} \cos^2(x)\ dx.$$

Answer:

$$\int_0^{\pi/2} \cos^2(x)\ dx = \frac{1}{2}(x+\sin(x)\cos(x))\Big|_0^{\pi/2} = \pi/4$$

(recall $\cos(\pi/2)=0$ and $\sin(0)=0$ – this means the only nonzero term comes from the x at the upper limit of integration).