MATH 136 – Calculus 2
Introducing the Integral – Total Accumulation and Area Problems
August 31, 2016

Background

Our course will study the second part of the subject of calculus: *integral calculus.* The first part, differential calculus, starts with a function and studies the instantaneous rate of change (the derivative). This part of calculus turns the tables around, so to speak, and asks questions like the first one below, where we know the rate at which some quantity is changing, but we want to know the total accumulation or amount of that quantity over some period of time. As we will see, that is connected with the problem of finding the *area under a graph.*

Questions

(1) A heavy rainstorm in Portland, Maine in October 1996 resulted in record rainfall[1] The estimated rainfall rate, $R(t)$, in units of cm of rain per hour during the storm is given in the following table, where the t values are given in hours after midnight on October 21:

t hours	$0-2$	$2-4$	$4-9$	$9-12$	$12-20$	$20-24$
$R(t)$ cm/hr	0.5	0.3	1.0	2.5	1.5	0.6

(a) Estimate the total rainfall in Portland during this 24-hour period.

(b) Draw a graph that illustrates the computation you were doing, and that shows the total accumulation of rain can be visualized as an area.

(2) What are some of the basic shapes for which you know area formulas from high school mathematics?

(3) The rainfall rate in the situation in question (1) was certainly not constant over each of the time periods in the table – it was certainly changing continuously. If we had a really accurate minute-by-minute, or second-by-second reading of the rainfall rate, how could we estimate the total rainfall more closely?

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[1]That is, a record for Portland. It was nothing like as heavy as the rain totals from recent storms in Louisiana earlier in August 2016 that caused the devastating flooding that's still plaguing that part of the country. I wasn't able to find the same sort of data for those storms in this form, although there were apparently times in Baton Rouge on August 12 where the rainfall rate was something like 7.5 cm/hour(!) Many places in Louisiana had over 70 cm of rain over a 24-hour period – that's nearly 28 inches(!)