6, 4, 6, 3   Total: 19

(If you can't decide whether an answer is correct, flag it and I'll take a look.)

MATH 136 Problem Set 2, Part B - Solutions

§5.4/62. The area of the triangle comes by using the base length $b-a$, and the height

$$y\left(\frac{a+b}{2}\right) = \left(\frac{a+b}{2} - a\right)\left(b - \frac{a+b}{2}\right)$$
$$= \frac{b-a}{2} \cdot \frac{b-a}{2}$$
$$= \frac{(b-a)^2}{4} \quad 1$$

give part credit for indicated steps; don't be too picky!

so $\text{Area (triangle)} = \frac{1}{2}(b-a) \cdot \frac{(b-a)^2}{4} = \frac{(b-a)^3}{8}$ 1

the area under the parabola is

$$\text{Area (parabolic arch)} = \int_a^b (x-a)(b-x)\,dx \quad 1$$
$$= \int_a^b -x^2 + (b+a)x - ab\,dx$$
$$= \left.-\frac{x^3}{3} + (b+a)\frac{x^2}{2} - abx\right|_a^b \quad 1$$
$$= -\frac{b^3}{3} + (b+a)\frac{b^2}{2} - ab^2 + \frac{a^3}{3} - (b+a)\frac{a^2}{2} + a^2b$$
$$= -\frac{b^3}{3} + \frac{b^3}{2} + \frac{ab^2}{2} - ab^2 + \frac{a^3}{3} - \frac{a^2b}{2} - \frac{a^3}{2} + a^2b$$
$$= \frac{b^3}{6} - \frac{b^2a}{2} + \frac{ba^2}{2} - \frac{a^3}{6}$$
$$= \frac{(b-a)^3}{6} \quad 1$$

1

Then $\frac{(b-a)^3}{6} = \frac{4}{3} \cdot \frac{(b-a)^3}{8}$, so $\text{Area (parabolic arch)} = \frac{4}{3}\,\text{Area (triangle)}$.

§ 5.5/45. Note $\frac{d}{dx}\int_0^x f(t)\,dt = f(x)$ by "FTC, Part II"

So:

(a) $A(x)$ is increasing when $f(x) > 0$: $(0,4)$, $(8,12)$

(b) local max at $x = 4$, $x = 12$ (First derivative test)

local min at $x = 8$

(c) Inflection points where $A''(x) = f'(x)$ changes sign,

so roughly $x = 1.7,\ 5.7,\ 10$ ← the first 2 are only approximate anything within ±.2 of these is OK.

(d) A is concave up when $A''(x) = f'(x) > 0$, so

$(0, 1.7)$, $(5.7, 10)$; concave down when

$A''(x) = f'(x) < 0$, so $(1.7, 5.7)$, $(10, 12.5)$

1 point each: give ½ part credit if they don't give all the points or intervals ½ part credit for correct points/intervals w/ no explanation

§ 5.6/31

(a) consumer surplus $= \int_0^{q^*} D(q)\,dq - p^* q^*$ 1

(b) producer surplus $= p^* q^* - \int_0^{q^*} S(q)\,dq$ 1

(c) If $S(q) = \frac{q}{100} + 1$ and $D(q) = \frac{10}{q/100 + 1}$

we find q^* by setting $S(q) = D(q)$ and solving:

$$\frac{q}{100} + 1 = \frac{10}{\frac{q}{100} + 1}$$

when $\left(\frac{q}{100} + 1\right)^2 = 10$

so $\frac{q}{100} + 1 = \sqrt{10}$, and $q^* = 100(\sqrt{10} - 1)$

$\doteq 216.2$

then $p^* = S(q^*) = D(q^*) = \sqrt{10}$ $\left(\doteq \$3.16\right)$, so (2)

$$\text{Consumer Surplus} = \int_0^{100(\sqrt{10}-1)} \frac{10}{q/100+1}\,dq - \sqrt{10}\cdot\left(100(\sqrt{10}-1)\right)$$

$\left(\text{let } u = \frac{q}{100}+1 \quad du = \frac{1}{100}\,dq\right)$

$$= 1000\ln\left(\frac{q}{100}+1\right)\Big|_0^{100(\sqrt{10}-1)} - 1000 + 100\sqrt{10}$$

full credit for getting this far

$$= 1000\ln(\sqrt{10}) - 1000 + 100\sqrt{10}$$

$\left(\doteq \$467.52\right)$ (1) (Don't be too picky about small arithmetic errors, answers written differently, ...)

$$\text{Producer surplus} = \sqrt{10}\cdot 100(\sqrt{10}-1) - \int_0^{100(\sqrt{10}-1)} \frac{q}{100}+1\,dq$$

$$= 1000 - 100\sqrt{10} - \left(\frac{q^2}{200}+q\right)\Big|_0^{100(\sqrt{10}-1)}$$

full credit this far.

$$= 1000 - 100\sqrt{10} - \frac{\left(100(\sqrt{10}-1)\right)^2}{200} - 100(\sqrt{10}-1)$$

$\left(\doteq \$450.00\right)$ (1)

5.7 / 109.

(1) $\int_a^b \frac{1}{x}\,dx = \ln(x)\Big|_a^b = \ln(b) - \ln(a) = \ln(b/a)$ (log property)

But:

(1) $\int_1^{b/a} \frac{1}{x}\,dx = \ln(x)\Big|_1^{b/a} = \ln(b/a) - \ln(1) = \ln(b/a)$ since $\ln(1)=0$

$$\text{So } \int_{2^n}^{2^{n+1}} \frac{1}{x}\,dx = \int_1^{2^{n+1}/2^n} \frac{1}{x}\,dx = \int_1^2 \frac{1}{x}\,dx = \ln(2), \text{ all } n.$$

So all areas shown are equal.