2, 2, 4, 4

MATH 136 PS 13 Solutions

§ 5.1 /

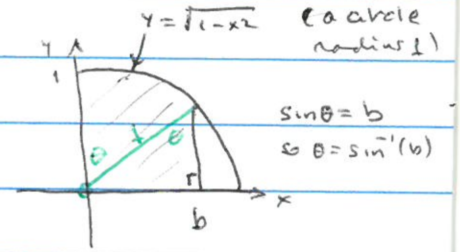
24. this can be written as: $\sum_{j=2}^{5} (j^2+j)$ (2)

26. this is: $\sum_{j=1}^{n} \sqrt{j+j^3}$ (2)

64. this is the limit of the right-hand Riemann sums for $f(x)=x^4$ on $[a,b]=[2,5]$. It represents the area between $y=x^4$ and the x-axis, for $2 \le x \le 5$. (4)

§ 5.2 /

86. $\int_0^b \sqrt{1-x^2}\,dx$ represents the area



Draw the radius from $(0,0)$ to $(b, \sqrt{1-b^2})$. the part of the shaded area below that radius is a right triangle with base b, height $\sqrt{1-b^2}$. Its area is $A_{triangle} = \frac{1}{2} b \sqrt{1-b^2}$. the part of the shaded region above the radius is a circular sector. the angle $\theta = \sin^{-1}(b)$, and the radius is 1, so the area is $A_{sector} = \frac{1}{2}\theta = \frac{1}{2}\sin^{-1}(b)$. the triangle and the sector only overlap along a line so

$$A_{total} = A_{triangle} + A_{sector}$$

$$= \frac{1}{2} b\sqrt{1-b^2} + \frac{1}{2}\sin^{-1}(b)$$

§ 5.3/70

By the chain rule and the derivative rules for $\sec x$, $\tan x$:

$$F'(x) = 2\tan x \cdot \sec^2 x$$

$$G'(x) = 2\sec x \cdot \sec x \cdot \tan x = 2\tan x \sec^2 x$$

this says $F(x) = G(x) + C$ for some constant C.

In fact from $\sin^2 x + \cos^2 x = 1$ for all x, dividing by $\cos^2 x$, we get $\tan^2 x + 1 = \sec^2 x$, so

$\boxed{G(x) = F(x) + 1}$.