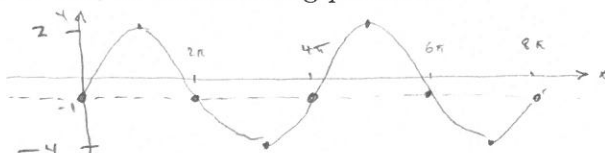


MATH 135 – Calculus 1  
 Practice on Trigonometric Functions  
 September 9, 2019

Background

In the video and class today, we have seen a “lightning review” of trigonometry. To practice on some ideas related to this, do the following problems.

Questions



$y = 3 \sin\left(\frac{x}{2}\right) - 1$   
 stretched vertically by 3 shifted down by 1  
 horizontally by 2

1) Sinusoids

- (a) Starting from the graph  $y = \sin(x)$  and using scaling and shifting, sketch the graph  $y = 3 \sin\left(\frac{x}{2}\right) - 1$  on the interval  $[0, 8\pi]$ . (Do not use a graphing calculator to generate your plot. You may check your work after you are finished, but try to do this by hand.)
- (b) The graph in part (1) is an example of a *sinusoid* (or sine-wave graph). The *amplitude* of a sinusoid is one-half the vertical distance between the minimum and maximum values. What is the amplitude of your sinusoid in part (a)?  $A = 3$
- (c) The *period* of a sinusoid  $f(x)$  is the *smallest strictly positive number*  $T$  for which it is true that  $f(x + T) = f(x)$  for all  $x$ . For example the period of  $f(x) = \cos(x)$  is  $T = 2\pi$ . What is the period of the sinusoid from part (a)?  $T = 4\pi$
- (d) Give a formula defining a sinusoidal graph  $y = f(x)$  with amplitude  $A = 7$  and period  $T = 5\pi$ , whose value at  $x = 0$  is  $f(x) = 0$ .  $y = 7 \sin\left(\frac{2x}{5}\right)$  is one (many others)

2) *By hand* (not using a graphing calculator) sketch the portion of the graph  $y = \cot(x) = \frac{\cos(x)}{\sin(x)}$  for  $0 < x < \pi$ , as follows:

- (a) First, determine where the vertical asymptotes are located and sketch the four closest to  $x = 0$ .  $x = 0, \pm\pi, \pm 2\pi, \dots$  (all  $x$  where  $\sin(x) = 0$ )
- (b) Next, mark the points on the intervals between your asymptotes where  $\cot(x) = 0$ .  $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots$  where  $\cos(x) = 0$
- (c) Starting from  $x = 0$ , is  $\cot(x)$  increasing or decreasing? It's decreasing up to  $x = \pi/2$  since  $\cos(x)$  is decreasing and  $\sin(x)$  is increasing.
- (d) Put everything together to sketch your graph.

3) How are  $\tan\left(x - \frac{\pi}{2}\right)$  and  $\cot(x)$  related?

$\tan\left(x - \frac{\pi}{2}\right) = -\cot(x)$

(can see this from graph, or with trig identities)

$\sin\left(x - \frac{\pi}{2}\right) = -\cos(x)$   
 $\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$

