

MATH 135 – Calculus 1
Practice on Linear and Quadratic Functions
September 6, 2019

Background

Every line in the plane is described by an equation of the form $Ax + By + C = 0$ for some constants A, B, C . If the line is not vertical ($B \neq 0$), then it is the graph of a function. (Why?) Recall that we say a function f is *linear* if $f(x) = mx + b$ for some constants m, b . The number m is called the *slope of the line* and the constant b is called the *y-intercept* of the line. *Quadratic functions* have the form $f(x) = Ax^2 + Bx + C$ where $A \neq 0$.

Questions

1) Consider lines with equations of the form $2x + cy - 3 = 0$.

$$y = -\frac{2}{c}x + \frac{3}{c}$$

- (a) For which value of c does the line contain the point $(x, y) = (1, 2)$? $c = \frac{1}{2}$
- (b) For which value of c does the line have slope -5 ? $c = \frac{2}{5}$
- (c) Is there any value of c such that the line is horizontal? Why or why not? no: $\frac{2}{c} \neq 0$
- (d) For which value of c is the line perpendicular to the line given by $5x - 3y + 1 = 0$? (Hint: What is true about slopes of perpendicular lines?) $c = \frac{10}{3}$

2) The volume V (in liters) of sample of 3 grams of carbon dioxide at 27 degrees Celsius was measured as a function of the pressure p (in atmospheres) with the results in the following table:

p	0.25	1.00	2.50	4.00	6.00
V	6.72	1.68	0.67	0.42	0.27

Is V (approximately) a linear function of p ? Why or why not? If so, find an approximate formula $V = mp + b$. If not, can you see a equation of a different form for V as a function of p ?

NO! slopes are not all equal: $\frac{6.72 - 1.68}{.25 - 1.00} = -6.72$ $\frac{0.42 - 0.27}{4 - 6} = \frac{.15}{-2}$

(3) By the algebraic technique of *completing the square*, every quadratic function $f(x) = x^2 + Bx + C$ (note the coefficient $A = 1$) can be rewritten in the form

$$f(x) = \left(x + \frac{B}{2}\right)^2 + \frac{4C - B^2}{4}$$

- (a) What does this tell us about the relation between the graphs $y = f(x)$ and $y = x^2$? Describe in words. *Shift horizontally + vertically*
- (b) Now suppose $A \neq 1$. What is true about the graph $y = Ax^2 + Bx + C$ when $A > 0$? What about when $A < 0$?

parabola opens up $A > 0$:
down $A < 0$: