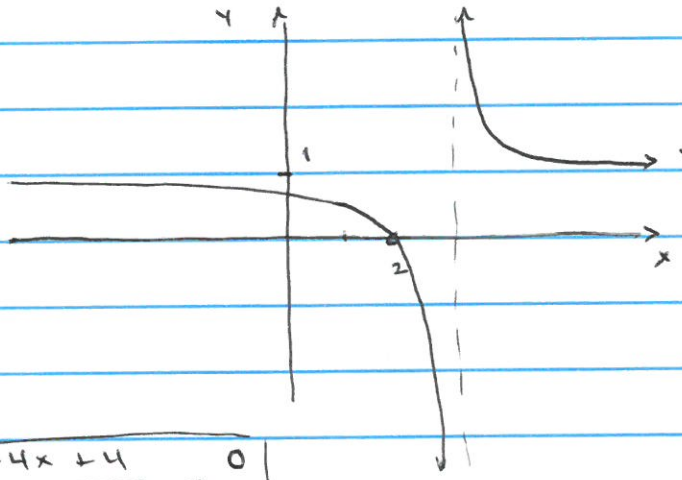
Solutions for Discussion Sheets 9/23-25:

A

(1) (a)



Guess:

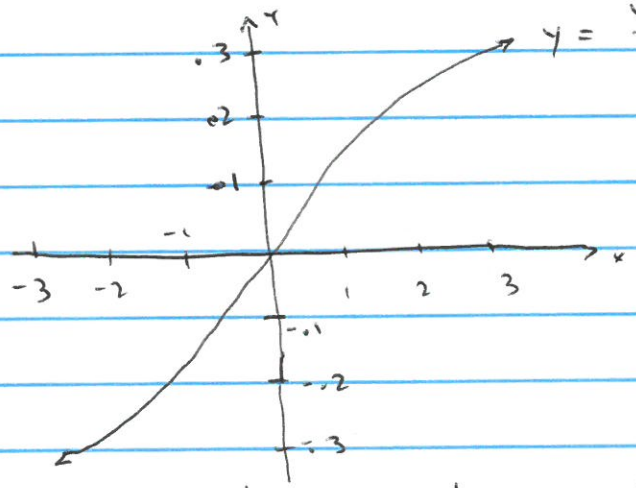
$$\lim_{x\to 2} \frac{x^2-4x+4}{x^2-5x+6} = 0$$

(b) $\lim_{x\to 2} x^2-5x+6 = 0$, so the limit quotient law does not apply

(c) $$\frac{x^2-4x+4}{x^2-5x+6} = \frac{(x-2)^2}{(x-2)(x-3)} = \frac{x-2}{x-3} \quad \text{if } x \neq 2$$

Now $\lim_{x\to 2} x-3 \neq 0$, and $\lim_{x\to 2} \frac{x-2}{x-3} = 0$.

(2) (a)



Guess:

$$\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x} = 0$$

(b) Same as in (1) above: $\lim_{x\to 0} x = 0$ in denominator.

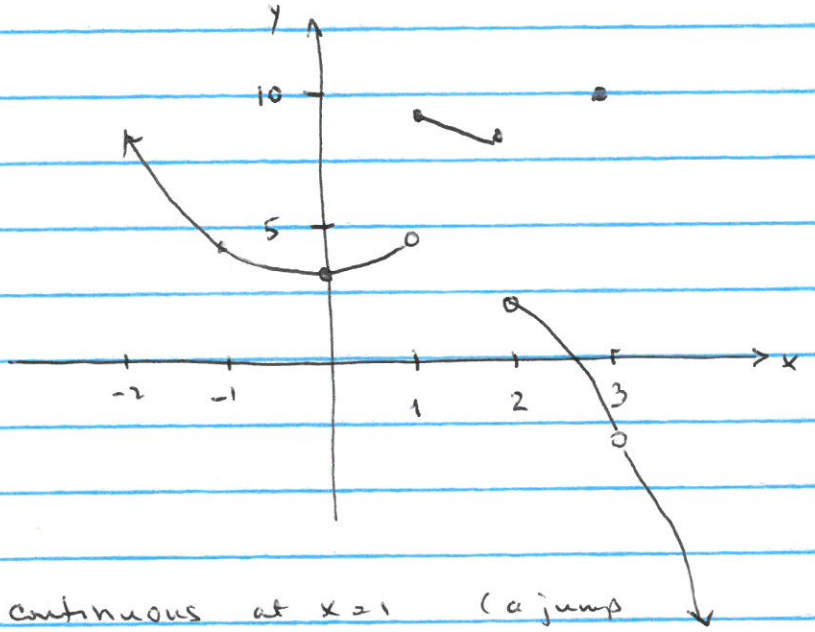
(c) $$\lim_{x\to 0} \frac{(\sqrt{x^2+9}-3)}{x} \cdot \frac{(\sqrt{x^2+9}+3)}{(\sqrt{x^2+9}+3)} = \lim_{x\to 0} \frac{x^2+9-9}{x(\sqrt{x^2+3}+3)}$$

$$= \lim_{x \to 0} \frac{x}{\sqrt{x^2+9} + 3}$$

(cancelling one factor of x top and bottom)

$= 0$. ✓

(B) (1)



(a) not continuous at $x=1$ (a jump discontinuity)

(b) not continuous at $x=2$ (jump discontinuity)

(c) not continuous at $x=3$ (removable discontinuity)

(d) $x=4$: continuous

(2) (a) not continuous — might have a price change (jumps)

(b) continuous

(c) not continuous - changes by jumps

(d) with arbitrary precision mercury thermometer, continuous, not with digital thermometer no

(3) infinite discontinuities at $x = \pm 4$ $(C=4)$

$\lim_{x \to 4^+} f(x) = -\infty$ $\lim_{x \to 4^-} f(x) = +\infty$

$\lim_{x \to -4^-} f(x) = -\infty$ $\lim_{x \to -4^+} f(x) = +\infty$

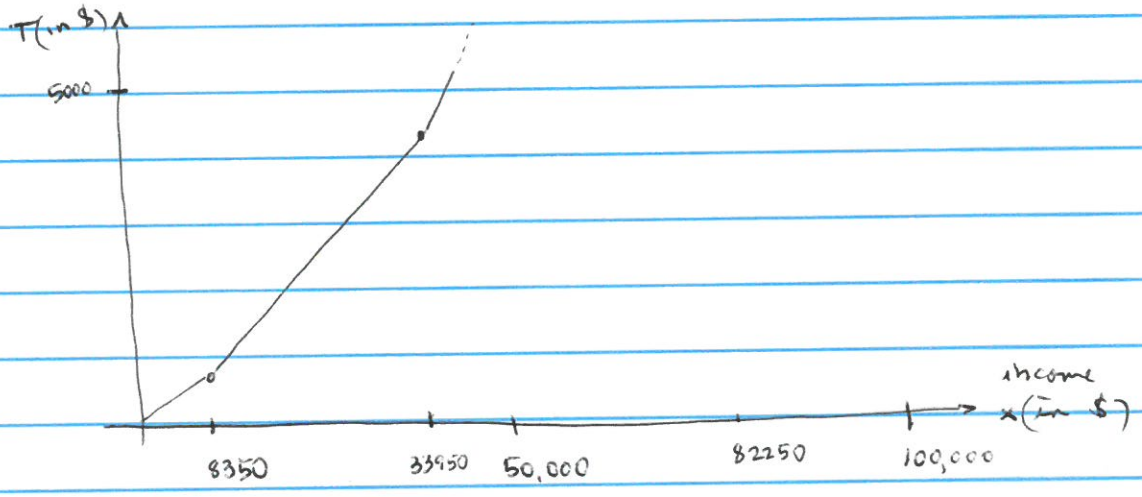
(4) $\lim_{x \to 8350^-} T(x) = \$ 835.00 \quad (= (.1) \cdot (8350))$

$\lim_{x \to 8350^+} T(x) = (.15)(8350) - 417.50 = \$ 835.00$

$\lim_{x \to 33950^-} T(x) = (.15)(33950) - 417.50 = \$ 4675.00$

$\lim_{x \to 33950^+} T(x) = (.25)(33950) - 3812.50 = \$ 4675.00$

So $T(x)$ is <u>continuous</u> for all $x > 0$.



In terms of taxes owed, increasing income <u>always</u> increases taxes due. So from the point of view of tax liability, it <u>isn't</u> ever advantageous to earn less money.