MATH 135 – Calculus 1
Limits Numerically and Graphically
September 18, 2019

Background

The tables of values of average velocities that we were looking at last time are aimed at trying to estimate values of instantaneous velocities:

$$v_{inst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}, \text{ or,} \tag{1}$$

and slopes of tangents:

$$m_{tan} = \lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}. \tag{2}$$

For the next few days we will be looking at this process of estimating limits. Eventually we will develop methods to compute limits exactly in many cases. To focus on the limiting process, we will momentarily take a step back from the particular limits in (1) and (2) and look at limits

$$\lim_{x \to c} f(x) \tag{3}$$

in general.

To start we will look at numerical and graphical methods to understand these general limits. Recall the general idea from our video for today – to say in (3) that $\lim_{x\to c} f(x) = L$, we want to be convinced that $|f(x) - L|$ can be made arbitrarily small by taking x sufficiently close to c, *but we won't just substitute $x = c$ because we want to allow for the limit to exist even when $f(c)$ does not exist.* (This is the situation for instance in (1) if we look at $\frac{x(t+\Delta t)-x(t)}{\Delta t}$ as a function of Δt – we want to all for a limit as $\Delta t \to 0$ to exist even though direct subsitution gives us 0/0, which is undefined(!) Similarly for (2), where the $\frac{f(a+\Delta x)-f(a)}{\Delta x}$ is viewed as a function of Δx.)

Questions

(1) Complete the following table and use your results to guess the limit $\lim_{x\to 4} \frac{x^2-16}{x-4}$. Use at least 5 decimal places in all calculations for these.

x	4.1	4.01	4.001	3.999	3.99	3.9
$\frac{x^2-16}{x-4}$	8.1	8.01	8.001	7.999	7.99	7.9

Looks like

$$\boxed{\lim_{x\to 4} \frac{x^2-16}{x-4} = 8}$$

(2) Complete the following table and use your results to guess the limit $\lim_{x\to 0} \frac{\cos(x)-1}{x}$ Use at least 5 decimal places in all calculations for these.

x	0.1	0.01	0.001	-0.001	-0.01	-0.1
$\frac{\cos(x)-1}{x}$	-.04996	-.005	-.0005	.0005	.005	-.04996

Looks like

$$\boxed{\lim_{x\to 0} \frac{\cos(x)-1}{x} = 0}$$

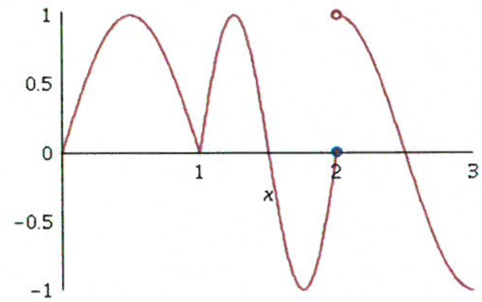

Figure 1: Figure for Question 4

(3) The limit in question (2) can be interpreted as a limit of the form from (2) before. What is the function $f(x)$ there? (Hint: It's not $\frac{\cos(x)-1}{x}$.) Does your guess of the value of the limit agree with what you would expect from the graph of $y = f(x)$ for your f?) It's $f(x) = \cos(x)$, $a = 0$
$x \leftrightarrow \Delta x$

(4) Refer to the graph above. What are the limits $\lim_{x\to 1^+} f(x)$, $\lim_{x\to 1^-} f(x)$, $\lim_{x\to 2^+} f(x)$, and $\lim_{x\to 2^-} f(x)$. Does either $\lim_{x\to 1} f(x)$ or $\lim_{x\to 2} f(x)$ exist? Why or why not?

$\lim_{x\to 1^+} f(x) = 0$ $\quad \lim_{x\to 1^-} f(x) = 0$ $\quad \lim_{x\to 1} f(x)$ exists, $= 1$.

$\lim_{x\to 2^+} f(x) = 1$ $\quad \lim_{x\to 2^-} f(x) = 0$ $\quad \lim_{x\to 2} f(x)$ DNE.