Key

MATH 135 – Calculus 1
Rates of Change, Slopes of Tangents
September 16, 2019

Background

We are now ready to move into Chapter 2 of our text and consider average rates of change, and eventually limits and tangent lines. *This is really the start of the subject of calculus itself.* Everything we have done up until now has been review of precalculus topics to prepare!

Questions

1) A ball dropped from a height of 40 meters and from a state of rest at time $t = 0$ has height $s(t) = 40 - 4.9t^2$ meters at time t seconds.

(a) How far does the ball travel between $t = 1$ and $t = 1.5$? What is the ball's average velocity over that interval? (Note: Your answer should be negative. That just means the ball is moving downward.) $s(1.5) - s(1) = 28.975 - 35.1 = -6.125$ $\quad v_{ave} = \frac{-6.125}{1.5-1} = -12.25$ m/sec

(b) Complete the following table by following the same steps you did in part (a) for each of the given intervals. Use at least 5 decimal places in all calculations for these.

interval	$[1, 1.1]$	$[1, 1.01]$	$[1, 1.001]$	$[1, 1.0001]$
ave.vel.	-10.29	-9.849	-9.8049	-9.8005

(c) Using this information, estimate the velocity of the ball *right at the instant* $t = 1$. This is called the *instantaneous velocity* of the ball at that time. $\boxed{v_{inst} \doteq -9.8 \text{ m/sec}}$

(d) Now repeat part (b) for these intervals *ending at* $t = 1$:

interval	$[0.9, 1]$	$[0.99, 1]$	$[0.999, 1]$	$[0.9999, 1]$
ave.vel.	-9.31	-9.751	-9.7951	-9.7995

Are these results consistent with what you did before in part (b)? (They should be!) Yes

looks like $\boxed{v_{inst} = -9.8 \text{ m/sec}}$

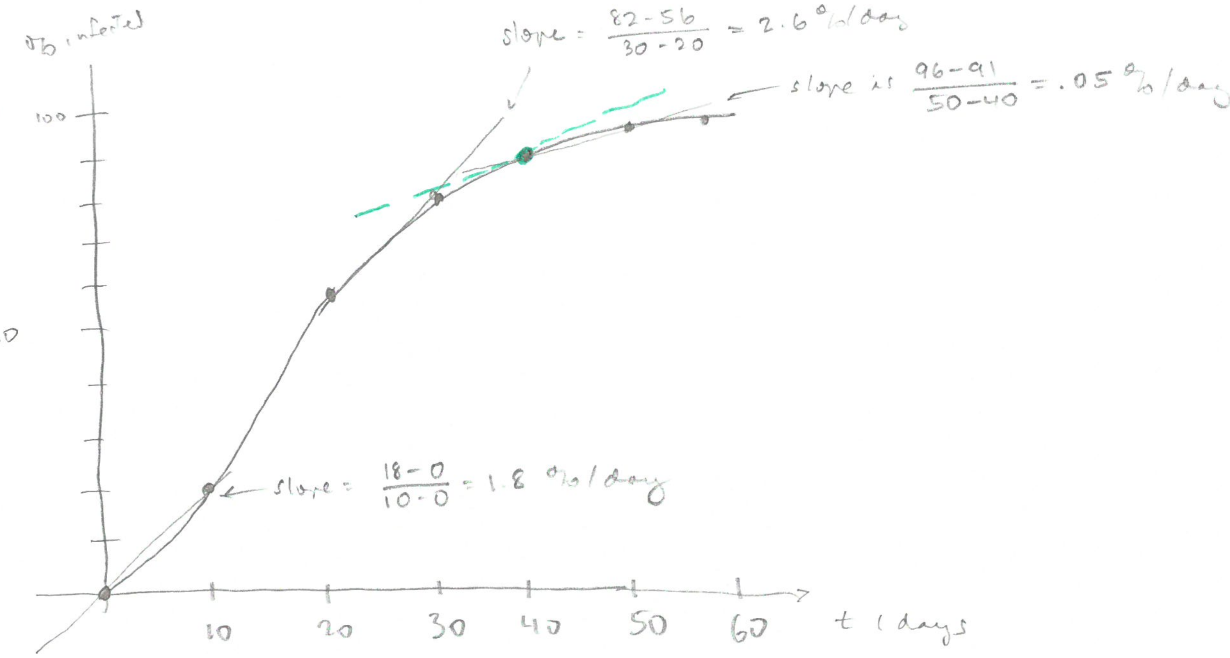
2) The fungus *Fusarium exosporium* infects a field of flax plants through the roots and causes the plants to wilt. Eventually the entire field is infected. The percentage $f(t)$ of infected plants as a function of time t (in days) is given in Table 1 (on the back of the page).

(a) Plot the points $(t, f(t))$ given in the table and connect to make a smooth curve.

(b) Use your graph to say which is largest, and which is smallest of
- the average infection rate over the time interval $[0, 10]$,
- the average infection rate over the time interval $[20, 30]$,
- the average infection rate over the time interval $[40, 50]$

(c) Using the table, compute the actual values of the average infection rates (in units of percent per day) over the intervals given in part (b).

(d) Draw a line tangent to your graph at $t = 40$. Estimate its slope from the information you have.

(a) 

(b+c) $[0,10]$: $\frac{18-0}{10-0} = 1.8$ %/day

$[20,30]$: $\frac{82-56}{30-20} = 2.6$ %/day ← largest

$[40,50]$: $\frac{96-91}{50-40} = .05$ % day ← smallest

$t =$ time in days	0	10	20	30	40	50	60
$f(t) =$ percent infected	0	18	56	82	91	96	98

Table 1: Percent of flax plants infected as function of time in days

(d) Slope of tangent at $t = 40$ can be estimated by

slope $[30,40]$: $\frac{91-82}{40-30} = .09$ % / day

or $[40,50]$: $.05$ % / day

or the average $\frac{.09 + .05}{2} = .07$ % / day