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MATH 135 4, 2, 2; 2 for first step. +2 EC for correct method ; 4 14 total

Problem Set 8, Part B Solutions

§4.3/68. Let $f(x) = Ax^2 + Bx + C$ be a quadratic polynomial. On any interval $[a,b]$:

$$\frac{f(b)-f(a)}{b-a} = \frac{(Ab^2+Bb+C)-(Aa^2+Ba+C)}{b-a} \quad 1$$

$$= A(b+a) + B \quad 1$$

If $c = \frac{a+b}{2}$, $f'(x) = 2Ax + B$, 1 so $f'(c) = \cancel{2}A\frac{(b+a)}{\cancel{2}} + B$. 1
Hence $f'(c) = \frac{f(b)-f(a)}{b-a}$, so the conclusion of the MVT is satisfied.

§4.4/22. If the graph is $y = f(x)$, the points of inflection of f are $\boxed{x = c}$ (just one of them). 2

24. If the graph is $y = f''(x)$, the points of inflection of f occur at $\boxed{x = a, d, f}$ where $f''(x) = 0$ and changes sign. 2

§4.5/54 $\lim\limits_{x \to 0^+} \ln(x) \tan^{-1}(x)$ is an $\infty \cdot 0$ indeterminate form

The eas(ier) way to do this is to rewrite as:

$$= \lim_{x\to 0^+} \frac{\ln(x)}{(\tan^{-1}(x))^{-1}} \qquad \frac{\infty}{\infty}$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-(\tan^{-1}(x))^{-2}\cdot\frac{1}{1+x^2}}$$

$$= \lim_{x\to 0^+} -\frac{(1+x^2)(\tan^{-1}(x))^2}{x} \qquad \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x\to 0^+} -\frac{(1+x^2)\cdot 2\tan^{-1}(x)\cdot\frac{1}{1+x^2} + 2x\left(\tan^{-1}(x)\right)^2}{1}$$

$$= 0 \qquad \text{since } \lim_{x\to 0^+} \tan^{-1}(x) = 0.$$

It's also possible to do this by rearranging in a "clever way" first, then using L'Hopital:

$$\lim_{x\to 0^+} \ln(x)\cdot\tan^{-1}(x) = \lim_{x\to 0^+} x\ln x \cdot \lim_{x\to 0^+}\frac{\tan^{-1}x}{x}$$

$$= \lim_{x\to 0^+}\frac{\ln(x)}{\frac{1}{x}}\cdot\lim_{x\to 0^+}\frac{\tan^{-1}x}{x} \qquad \frac{\infty}{\infty}\cdot\frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{x^2}}\cdot\lim_{x\to 0^+}\frac{\frac{1}{1+x^2}}{1}$$

$$= 0\cdot 1 = 0.$$

(Note: we will learn a much easier way to show this next semester ;)

§4.6/54

(a) is (D) vertical asymptotes at $x=\pm 1$, and $y=-1$ when $x=0$

(b) is (A) since $\lim_{x\to\pm\infty}\frac{x^2}{x^2+1} = 1$ (no other graph has a horizontal asymptote at $y=1$)

(c) is (B) since $\lim_{x\to\pm\infty}\frac{1}{x^2+1} = 0$ and no vertical asymptotes

(d) is (C) since there are 2 vertical asymptotes at $x=\pm 1$ and $y=0$ when $x=0$.

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