2, 2, 6, 4   Total: 14

MATH 135, PROBLEM SET 7, PART B   Solutions

§3.8/40 $y = \tan^{-1}\left(\frac{1+t}{1-t}\right)$, so by the chain rule:

$$y' = \frac{dy}{dt} = \frac{1}{1+\left(\frac{1+t}{1-t}\right)^2} \cdot \frac{(1-t)\cdot 1 - (1+t)\cdot(-1)}{(1-t)^2} \quad 2$$

If they stop here, say "simplify"

$$= \frac{2}{(1-t)^2 + (1+t)^2}$$

$$= \frac{2}{2+2t^2}$$

$$= \frac{1}{1+t^2}$$

not necessary to simplify this far

they are not responsible for this

Comment: This should surprise you, since $\frac{d}{dt}\tan^{-1} t = \frac{1}{1+t^2}$ as well! The explanation is an identity for the tangent function: If $y = \tan^{-1}\left(\frac{1+t}{1-t}\right)$ then $\tan y = \frac{1+t}{1-t}$, and solving for t, we get

$$(1-t)\tan y = 1+t$$

so $$t(1+\tan y) = 1 - \tan y$$

$$t = \frac{1-\tan(y)}{1+\tan(y)}$$

$$= \frac{\tan(\frac{\pi}{4}) - \tan(y)}{1+\tan(\frac{\pi}{4})\tan(y)}$$

$$= \tan\left(y + \frac{\pi}{4}\right) \quad \text{(addition formula)}$$

Hence $y = \tan^{-1}(t) - \frac{\pi}{4}$, and the derivatives are equal as a result.

§3.9/78

1 $\frac{d}{dx}\ln(2x) = \frac{1}{2x}\cdot\frac{d}{dx}(2x) = \frac{1}{2x}\cdot 2 = \frac{1}{x} = \frac{d}{dx}\ln(x)$

The explanation comes from a property of logarithms.

1 Recall $\ln(A\cdot B) = \ln(A) + \ln(B)$ for all $A, B > 0$

Hence $\boxed{\ln(2x) = \ln(2) + \ln(x)}$, the derivatives are equal because $\ln(2x)$ and $\ln(x)$ differ by an additive constant.

§4.2/ If $h(t) = (t^2-1)^{1/3}$, then $h'(t) = \frac{1}{3}(t^2-1)^{-2/3}\cdot 2t$ 1

or $h'(t) = \frac{2}{3}\cdot\frac{t}{(t^2-1)^{2/3}}$

Critical points: $t=0$ where $h'(t)=0$

$t=\pm 1$ where $h'(t)$ DNE 2

(this is consistent with Figure 17 since the tangent lines at $t=\pm 1$ are vertical, while the tangent line at $t=0$ is horizontal.) 1

on $[0,1]$: $h(0) = -1$ is the minimum

$h(1) = 0$ is the maximum 1

on $[0,2]$: $h(0) = -1$ is the minimum

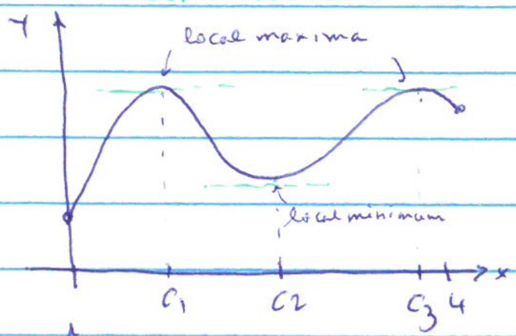
$h(1) = 0$ 1

$h(2) = \sqrt[3]{3} \doteq 1.44$ is the maximum

Note: Any graphs that satisfy the stated conditions are OK. If you're not sure, don't grade and leave me a note to check the answer.

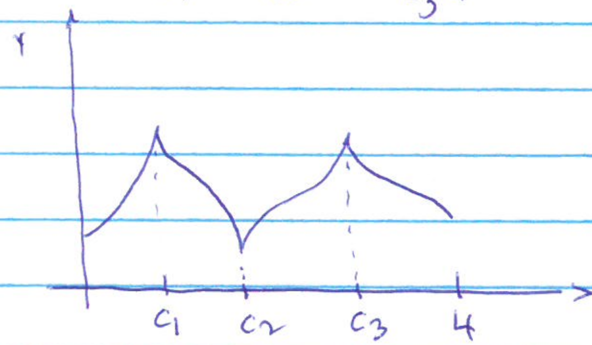
88.

(a)



is one such graph where $f'(c_i) = 0$

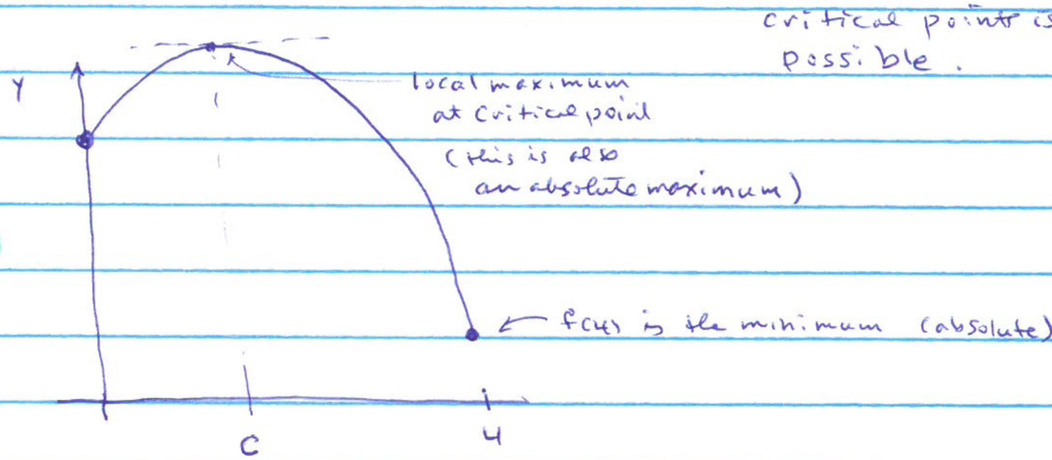
2



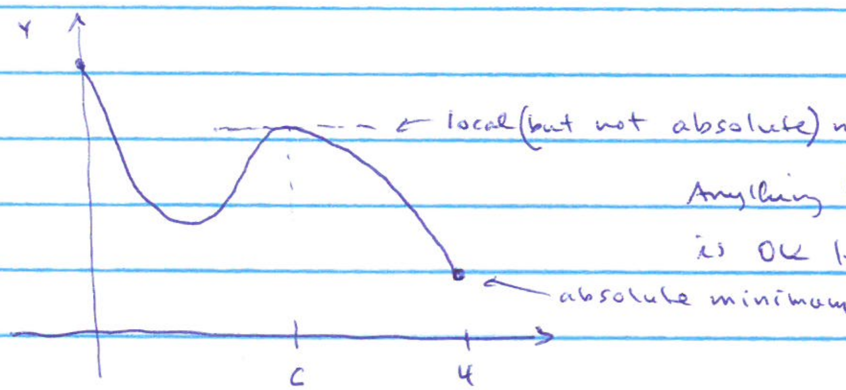
would be another where $f'(c_i)$ DNE

Any combination of the two types of critical points is also possible.

(b)



2



Anything like this is OK too